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A multi-Component Repairable System With Spares and State-sependent Rates

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Abstract: The queue size distribution at equilibrium for a machining system having M operating units along with S spares is obtained. There are R permanent repairmen available which provide the repair to the units failed while operating according to *FCFS*-discipline. The life times and repair times of all the units are assumed to be negative exponentially distributed with rate depending on the existing workload. Since the reliability of the system depends upon the system configuration. Therefore the provision of r special additional repairmen, which turn on according to a threshold rule depending upon the failed units in the system, is also made. This will help in reducing the backlog in case of long queue. Expression for the number of failed units in system is derived by using steady-state queue size distribution. Some other system characteristics are also reported a heuristic approach is facilitated to obtain the optima number of repairmen and spares simultaneously by minimizing the cost function

Key words: Markov, Standby, Machine Repair, Reliability, State-dependent Rates, Cost Function, Queue, Additional Repairmen.

1. Introduction

In the recent years, the advanced technology has been developed such that the system designer may meet out the desired demand of production by improving the reliability and availability of the system. Maintenance has a major impact over a long run. As soon as the system becomes more sophisticated and its units become more interdependent, the impact will increase. In view of such a design, the standby units play an important role so that the system may keep working to provide the desired grade of service all the time. The standby unit may replace the failed unit whenever the operating unit breaks down. In many applications, the behaviour of the failed unit to join the queue may depend upon the number of failed units. When all the spares are used and all permanent repairmen are busy and a unit breaks down, the production will be effected. Therefore, for maintaining continuous magnitude of the production, it is recommended that the special repair facility may be provided.

Many authors have reported research works in the field of machining system by using queue-theoretic approach. A cost model for cold standby was described by Hilliard [6]. M/M/C/m/m model with spares was studied by Gross and Harris [4]. A semi-numerical iterative method for solving a machine interference problem was given by Jain and Sharma [11]. Cherian et al. [1] studied the reliability of standby system with repair. Wang [19] provided the profit analysis of the machine repair problem with a single service station subject to breakdown. Optimal policies for

$$(1-t)^m \, C_m \left[a \, ; x\,(1-t)\right] = \sum_{n=0}^{\infty} \frac{(-m)_n}{n!} \; C_{m-n} \;\; (a\,; x)\, t^n$$

which can be well compared with a result of Mcbride by Truesdell method.

(ii) Again, setting $b = 1$, and $c = 0$ in (2.12) we obtain

$$e^t (1-t/x)^a \, C_m \left[a \, ; x - t\right] = \sum_{n=0}^{\infty} \frac{1}{n!} \, C_{m+n} \;\; (a\,; x)\, t^n$$

This was also obtained by Mcbride by Truesdell method.

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The Chapman-Kolmogorov balance equations governing the model are given by:

(20) $-N\lambda P_0 + \mu P_1 = 0$

(21) $-[N\lambda + n\mu]\, P_n + N\lambda\, P_{n-1} + (n+1)\,\mu\, P_{n+1} = 0 \qquad 1 \le n < S$

(22) $-[(N+S-n)\lambda + n\mu]\, P_n + [(N+S-n+1)\lambda\,]\, P_{n-1} + (n+1)\mu\, P_{n+1} = 0, \quad S \le n < R$

(23) $$-\left[(N+S-R)\left(\frac{R}{n+1}\right)^{b}\lambda + R\mu\right] P_R + (N+S-R+1)\lambda\, P_{n-1} + \left(\frac{R+1}{R}\right) R\,\mu\, P_{R+1} = 0,$$

(24) $$-\left[(N+S-n)\left(\frac{R}{n+1}\right)^{b}\lambda + \left(\frac{n}{R}\right)^{a} R\mu\right] P_n + (N+S-n+1)\left(\frac{R}{n}\right)^{b}\lambda\, P_{n-1} + \left(\frac{n+1}{R}\right) R\mu\, P_{n+1} = 0 \qquad R \le n < T$$

(25) $$-\left[(N+S-T)\left(\frac{R+1}{T+1}\right)^{b}\lambda + \left(\frac{T}{R}\right)^{a} R\mu\right] P_T + (N+S-T+1)\left(\frac{R}{T}\right)^{b}\lambda\, P_{T-1} + \left(\frac{R+1}{R+1}\right)^{a}(R\mu + \mu_1)\, P_{T+1} = 0,$$

(26) $$-\left[(N+S-n)\left(\frac{R+m}{n+1}\right)^{b}\lambda + \left(\frac{n}{R+m}\right)^{a}(R\mu + \mu_m)\right] P_n + (N+S-n+1) \left(\frac{r+m}{n}\right)^{b}\lambda\, P_{n-1} + \left(\frac{n+1}{R+m}\right)^{a}(R\mu + \mu_m)\, P_{n+1}\,\Bigg]\, P_{n+1} = 0.$$

$MT < n < (m+1)T, \quad 1 \le m < r$

(27) $$-\left[(N+S-n)\left(\frac{R+r}{n+1}\right)^{b}\lambda + \left(\frac{n}{R+r-1}\right)^{a}(R\mu + \mu_{r-1})\right] P_n + (N+S-n+1)\left(\frac{R+r-1}{n}\right)^{b}\lambda\, P_{n-1} + \left(\frac{R+1}{R+r}\right)(R\mu + \mu_1\Big]\, P_{n+1} = 0. \qquad N = rT$$

(28) $$-\ (N+S-n)\left(\frac{R+r}{n+1}\right)^{b}\lambda + \left(\frac{n}{R+r}\right)^{a}(R\mu + \mu_1)\Big]\, P_n + (N+S-n+1)\left(\frac{R+r}{n}\right)^{b}\lambda\, P_{n-1} + \left(\frac{n+1}{R+r}\right)^{a}(R\mu + \mu_1)]\, P_{n+1} = 0. \qquad rT < n < S+K$$

(29) $$-\left(\frac{S+K}{R+r}\right)^{a}(R\mu + \mu_r)]\, P_{S+K} + (N-K+1)\left(\frac{R+r}{S+K}\right)^{a}\lambda\, P_{S+K-1} = 0,$$

We find the probabilities for different states by solving the equations (20)-(29) as :

$$(30)\quad P_n=\begin{cases} \dfrac{N^n \rho^n}{n!} P_0 & 0<n\le S \\ \dfrac{N^n N!\,\rho^n}{n!(N+S-n)!} P_0, & S<n\le R \\ \dfrac{N^S N!\,\rho^n}{R!(N+S-n)!\,R^{(1-D)(n-R)}\left(\frac{n!}{R!}\right)^D} P_0, & R<n\le T \\ \dfrac{N^S N!\,\rho^n}{R!(N+S-n)!\,R^{n-(1-D)R-DT}\left(\frac{n!}{R!}\right)^D} \prod_{k=1}^{m-1}\left\{\dfrac{(R+k)^{DT}}{\left(1+\frac{\mu_k}{R_\mu}\right)^T}\right\} \dfrac{(R+m)^{(n-mT)D}}{\left(1+\frac{\mu_m}{R\mu}\right)^{(n-mT)}} P_0, & mT<n\le(m+1)T,\quad 1\le m<r \\ \dfrac{N^n N!\,\rho^n}{R!(N+S-n)!\,R^{n-(1-D)R-DT}\left(\frac{n!}{R!}\right)^D} \prod_{k=1}^{r-1}\left\{\dfrac{(R+k)^{DT}}{1+\left(1+\frac{\mu_k}{R\mu}\right)^T}\right\} \dfrac{(R+k)^{(n-rT)D}}{\left(1+\frac{\mu_k}{R\mu}\right)^{(n-rT)}} P_0, & rT<n\le S+K \end{cases}$$

The normalizing condition (15) results in

$$(31)\quad P_0^{-1}=\sum_{n=0}^{S}\frac{N^n\rho^n}{n!}+\sum_{n=S+1}^{R}\frac{N^S N!\,\rho^n}{n!(N+S-n)}+\frac{N^S N!}{R!}\sum_{n=R-1}^{T}\frac{\rho^n}{(N+S-n)R^{(1-D)(n-R)}\left(\frac{n!}{R!}\right)^D}+$$

$$+\frac{N^n N!}{R!}\sum_{n=mT+1}^{(m+1)T}\frac{\rho^n}{(N+S-n)R^{n-(1-D)R-DT}\left(\frac{n!}{R!}\right)^D}\prod_{k=1}^{m-1}\left\{\frac{(R+k)^{DT}}{\left(1+\frac{\mu_k}{R\mu}\right)^T}\right\}\frac{(R+m)^{(n-mT)D}}{\left(1+\frac{\mu_r}{R\mu}\right)^{(n-DT)}}$$

$$+\frac{N^n N!}{R!}\sum_{n=rT+1}^{S+K}\frac{\rho^n}{(N+S-n)!\,R^{n-(1-D)R-DT}\left(\frac{n!}{R!}\right)^D}\prod_{k=1}^{r-1}\left\{\frac{(R+k)^{DT}}{\left(1+\frac{\mu_k}{R\mu}\right)^T}\right\}\frac{(R+r)^{(n-rT)D}}{\left(1+\frac{\mu_r}{R\mu}\right)^{(n-rT)}}$$

The average number of failed units in the system

$$(32)\quad E(N)=\sum_{n=0}^{S+K} n\,P_n$$

$$=\sum_{n=0}^{S}\frac{N^n\rho^n}{(n-1)!}P_0+\sum_{n=S+1}^{R}\frac{N^n\rho^n n}{n!(N+S-n)!}P_0+$$

$$+\frac{N^S N!}{R!}\sum_{n=R+1}^{r}\frac{n\rho^n}{(N+S-n)R^{(1-D)(n-R)}\left(\frac{n!}{R!}\right)^D}P_0$$

$$+\frac{N^n N!}{R!}\sum_{n=mT+1}^{(m+1)T}\frac{n\rho^n}{(N+S-n)R^{n-(1-D)r-DT}\left(\frac{n!}{R!}\right)^D}\prod_{k=1}^{m-1}\left\{\frac{(R+k)^{DT}}{\left(1+\frac{\mu_k}{R\mu}\right)^T}\right\}\frac{(R+m)^{(n-mT)D}}{\left(1+\frac{\mu_m}{R\mu}\right)^{(n-mT)}}P_0$$

$$+\frac{N^n N!}{R!}\sum_{n=rT+1}^{S+K}\frac{n\rho^n}{(N+S-n)!\,R^{n-(1-D)R-DT}\left(\frac{n!}{R!}\right)^D}\prod_{k=1}^{r-1}\left\{\frac{(R+k)^{DT}}{\left(1+\frac{\mu_k}{R\mu}\right)^T}\right\}\frac{(R+r)^{(n-rT)D}}{\left(1+\frac{\mu_r}{R\mu}\right)^{(n-rT)}}P_0$$

4. Some System Characteristics

Using steady-state queue size distribution given equations (13) and (30) for case I and II, we also obtain some more performance measures as follows:

- The average number of spare units in the system is obtained as

(33) $$E(S)=\sum_{n=0}^{S}(S-n)P_n$$

- The average number of operating units in the system is given by

(34) $$E(0)=N-\sum_{n=Y+1}^{S+K}(n-S)P_n$$

- The average number of permanent idle repairmen is given by

(35) $$E(I)=\sum_{n=0}^{R-1}(R-n)P_n$$

The average number of permanent busy serves in the system is

(36) $$E(B)=R-E(I)$$

- The rate of production per unit is given as

(37) $$\text{P.R.}=1-\frac{E(N)}{N+S}$$

- The operating utilization is denoted as

(38) $$\text{O.U.}=\frac{E(B)}{R+r}$$

5. Cost Functions

Our main aim in this section is to provide a cost function, which is to be minimized to determine the optimal number of repairmen and spares by considering different costs. The total average cost is given by :

(39) $$E(C)=C_E\sum_{n=S+}^{S+K}(n-S)P_n+C_SE(S)+C_IE(I)+C_BE(B)+C_A\sum_{n=T+I}^{S+K}(n-T)P_n$$

where,

C_E Cost per unit time when all spares are employed.
C_S Cost per unit time for providing one spare unit.
C_B Cost per unit time when one permanent repairman is in busy state.
C_I Cost per unit time when one permanent repairman is in idle state.
C_A Cost per unit time of providing one additional repairman.

We can illustrate minimum cost as follows:

$$\text{Min }(Z^*)=F(R^*.S^*).$$

subject to constraints $A_v=\sum_{n=0}^{S}P_n\ge A$.

Here Av denotes the availability of the system while A shows the minimum fraction of time. Since analytical solution for evaluating the optimal number of repairmen and spares is very difficult, a direct research technique may be employed to determine the optimum value of the cost.

6. Discussion

In this paper we have tackled the reliability issues of a multi-component system consisting of S warm standby spares along with R permanent repairmen and r additional repairmen so that the system may provide regular magnitude of production up to a desired grade of demand. The steady-state probability distribution of the average number of breakdown units and some system characteristics are developed. In the last section of the paper, a cost function is also facilitated that may be very useful to practitioners and other system designers for a stream of tasks in various manufacturing / production processes.

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