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A Note On Stokes Drag On Axi-symmetric bodies : A New Approach

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Abstract: In the recent paper [1], author have proposed a simple formulae for evaluating the axial and transverse Stokes drag on axially symmetric bodies. Continuing the efforts in this regard, the axial and transverse drag forces have been evaluated for the cassini body, hypo-cycloidal body and cylindrical capsule with circular cross-section of radius 'b' and semispeherical caps on both ends. Further, the moments on these bodies have also been calculated.

Keywords: Stokes drag, axially symmetric body, cassini body, hypocycloidal body, cylindrical capsule.

Introduction:

In the recent paper [Datta and Srivastava, 1999, 1], authors have proposed a simple formulae, based on the integral [p.122,2] used to evaluate drag on a sphere, for finding the axial and transverse Stokes drag on axi-symmetric bodies.

The axial flow

The drag on body, when it is situated in axi-symmetric Stokes flow with uniform stream U_x along x-axis is given as [1].

(1.1) $$F_x = \frac{1}{2}\frac{\lambda(y_{\max.})^2}{h},$$

where

(1.2) $$\lambda = 6\pi\mu U_x$$

and

(1.3) $$h = \left(\frac{3}{8}\right)\int_0^{\pi} R\sin^3\alpha \, d\alpha$$

Here, R is the intercepting length between the point on the meridional curve and axis of symmetry (x-axis) of the body and α is the slope of normal [See figure 1]. In cartesian coordinates, h can be expressed as

(1.4) $$h = \left(-\frac{3}{4}\right)\int_0^{a} \frac{yy''}{(1+y'^2)^2}\,dx,$$

where, $x = a$ is the maximum axial length and dashes represents derivatives with respect to x.

The transverse flow

Let us consider an axially symmetric body [see figure 1] placed in a uniform stream U_y along transverse axis (y-axis). The Stokes drag on this body is given to be [1]

$$F_y = \frac{1}{2} \cdot \frac{\lambda (y_{max.})^2}{h_y}, \tag{1.5}$$

where

$$\lambda = 6\pi \mu U_y$$

and

$$h_y = \frac{3}{16} \int_0^{\pi} (2R \sin\alpha - \sin^3\alpha)\, d\alpha, \tag{1.6}$$

in cartesian coordinates, it can be expressed as

$$h_y = \left(\frac{3}{8}\right) \int_0^{a} \left[\frac{yy''}{(1+y'^2)^2} - \frac{yy''}{(1+y'^2)^2} \right] dx. \tag{1.7}$$

The Moment

The moment on the axially symmetric body rotating slowly with uniform angular velocity Ω about axis of symmetry is given as [1]

$$M_x = \left(\frac{2}{3}\right) \left[\frac{(y_{max.})^2 \Omega}{U} \right] F_x, \tag{1.8}$$

where, F_x is the axial drag on body. The result (1.1, 1.5, 1.8) have already been used for the bodies (viz; sphere, spheroid, deformed sphere, cycloidal body of revolution and egg-shaped body) and are given paper [1]. Now, in this continuation, these results have been used for cassini body of revolution, hyp-ocycloidal body of revolution and cylindrical capsule having semi-spherical caps with same radius. It has been found here that the results of Stokes drag and moments are new and never existed in the literature.

2. Flow past cassini body of revolution

Let us consider the cassini body (figure 2) obtained by revolving the curve

$$y^2 = \left(\frac{2}{3}\right)(1+3x^2)^{\frac{1}{2}} - x^2 - \frac{1}{3}, \quad 0 \le x \le 1, \tag{2.1}$$

about x-axis (axis of symmetry).

By using the (1.1) together with (1.4), the axial drag on cassini body will be, with $y_{max.} = 0.577$

(2.2) $$F_x \approx 0.8\lambda, \quad \lambda = 6\pi\mu U_x$$

and the transverse Stokes drag on cassini body can be easily obtained by using (1.5) together with (1.6), $y_{max.} = 0.577$

(2.3) $$F_y \approx 0.82\,\lambda, \quad \lambda = 6\pi\mu U_y$$

Also, by using (1.8), the moment on cassini body rotating with angular velocity Ω about axis of symmetry is given to be, $y_{max.} = 0.577$

(2.4) $$M_x \approx 1.066\ \pi\mu\,\Omega$$

3. Flow past hypocycloidal body of revolution

Let us consider the hypocycloidal body (figure 3) obtained by the curve

(3.1) $$y^2 = -3x^2 + (1+8x^4)^{\frac{1}{2}}, \qquad 0 \le x \le 1,$$

about axis of symmetry (x-axis).

By using (1.1) together with (1.4), $y_{max.} = 1.0$, the axial Stokes drag on hypocycloidal body will be given to be

(3.2) $$F_x \approx 1.044\,\lambda, \quad \lambda = 6\pi\mu U_x,$$

and the transverse Stokes drag, with same y, on this body can be obtained by using (1.5) and (1.6)

(3.3) $$F_y \approx 1.32\,\lambda, \quad \lambda = 6\pi\mu U_y$$

Also, the moment on hypocycloidal body rotating with angular velocity Ω about the axis of symmetry is given to be, with the help of (1.8)

(3.4) $$M_x \approx 4.176\,\pi\mu\,\Omega.$$

4. Flow past cylindrical capsule

Let us consider the cylindrical capsule (figure 4) with semi-spherical caps on both ends having same radius 'b', obtained by revolving the curves (PA, the circular segment, AA', the line segment, $A'P'$, again circular segment)

(4.1) $$\left.\begin{array}{lll} PA, & x = b\cos t, \; y = b\sin t, & 0 \le t \le \pi/2 \\ AA', & y = b, \quad \theta = \pi/2 & \\ A'P', & x = b\cos t, \; y = b\sin t, & \pi/2 \le t \le \pi \end{array}\right\}$$

about the axis of symmetry, x-axis.

By using (1.1) and $y_{max.} = b$, the axial drag on the capsule is comes out to be

(4.2) $$F_x = 6\pi\,\mu U_x b$$

and the transverse drag on the body is given by using (1.5), $y_{max.} = b$,

$$F_y = 6\pi \mu U_y b. \tag{4.3}$$

Also, the results for moment on cylindrical capsule rotating with small angular velocity Ω about the axis of symmetry with $y_{\max} = b$, by using (1.8) comes out to be

$$M_x = 4\pi \mu b^3 \Omega.$$

These results (4.2, 4.3, 4.4) are same as to that of sphere, it may be happen due to the fact that straight line segment (AA') occurred in the meridional curve (4.1) which, does not contribute in the drag force.

It should be kept in mind, while using these results, that the numerical value of integral (1.4) involve the error of $o(h^3)$, due to the Simpson's one third rule, where 'h' is the step length. Therefore, all the results of drags and moments for various proposed axially symmetric bodies are in approximation, but to a valid limits.

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REFERENCES

[1] Datta, S. and Srivastava, D.K. "*Stokes drag on axially symmetric bodies: a new approach*" Proc. (Math.Sci.), Ind. Acad Sci., Vol. 109 No.4, Nov. 1999, 441–452..

[2] Happle, J. and Brenner, H. "*Low Reynolds Number Hydrodynamics*" Prentice Hall Englewood, 1983, 1965, Clifs NJ.

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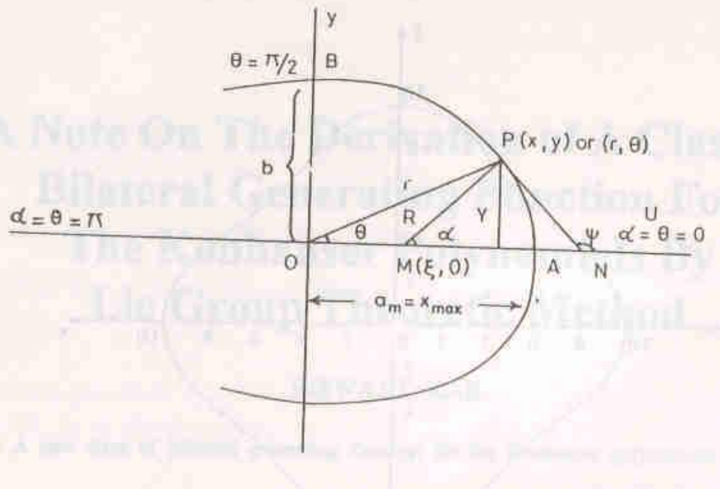

(PN, PM are tangent and normal)

Fig. 1 Geometry of axially symmetric body.

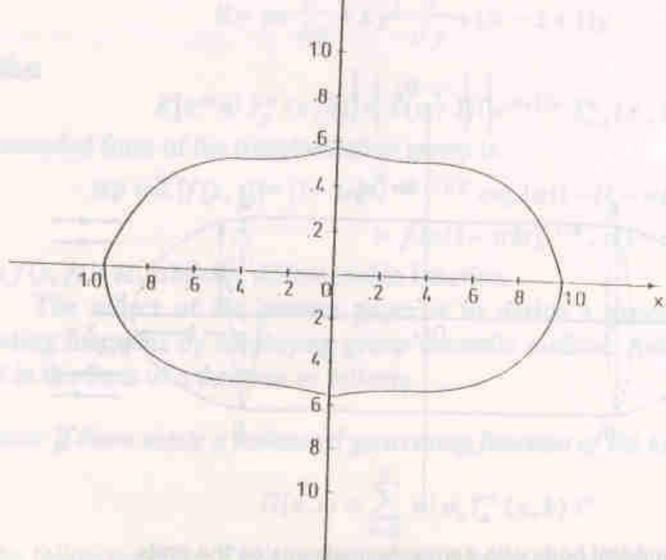

Fig. 2 Cassini curve (cassini body of revolution).

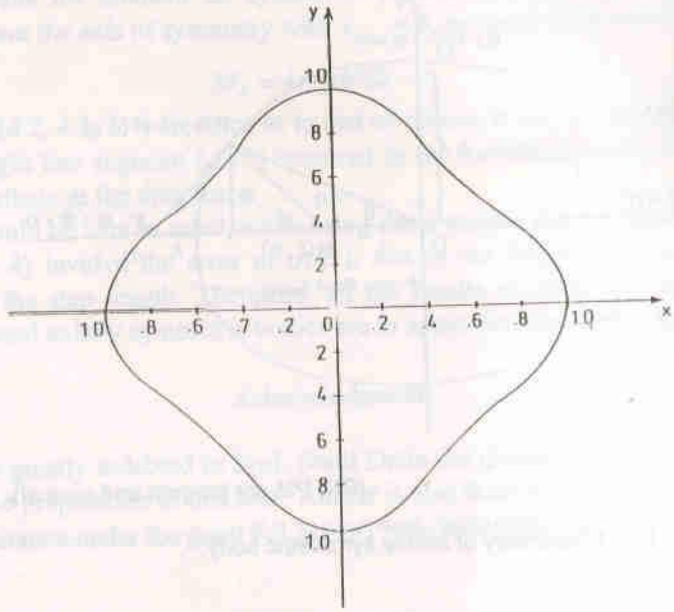

Fig. 3 Hypocycloidal like profile.

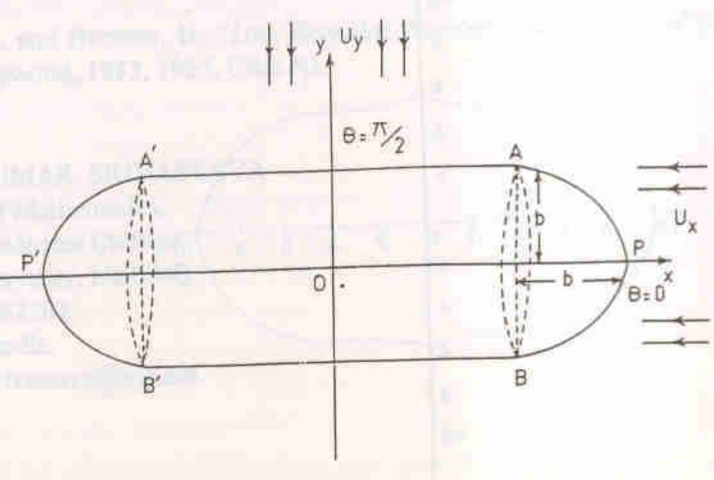

Fig. 4 Cylindrical body with semispherical caps on the ends.