The Nepali Math. Sc. Report
Vol. 23, No. 2, 2004.

A Note on Stokes Drag on Axi-symmetric Body : Oblique : Angle of Attack

DEEPAK KUMAR SRIVASTAVA

Abstract : In the paper [1], author have proposed a simple formulae for evaluating the axial and transverse Stokes drag on axi-symmetric bodies. Continuing the efforts in this regard, the expression for drag has been given when the axially-symmetric body is placed in the slow uniform incompressible viscous flow with oblique angle of attack 'ζ'. The drags in such flow have been calculated for sphere, spheroids (prolate and oblate), deformed sphere, cycloidal body, egg-shaped body, cassini body and hypocycloidal body.

Key words : Stokes drag, axially symmetric body, oblique angle, cycloidal body, egg-shaped body, cassini body, hypocycloidal body.

AMS Subject Classification : 76 D

1. Introduction

In the recent paper [Datta and Srivastava, 1999,1], authors have proposed a simple formulae, based on the integral [p. 122,2] used to evaluate drag on a sphere, for finding the axial and transverse Stokes drag on axi-symmetric bodies.

The Axial Flow

The drag on body, when it is placed in axi-symmetric Stokes flow with uniform stream U along x-axis is given as [1].

(1.1) $$F_x = \frac{1}{2}\frac{\lambda (y_{max})^2}{h},$$

where

(1.2) $$\lambda = 6\pi\mu U$$

and

(1.3) $$h = \left(\frac{3}{8}\right) \int_{\alpha=0}^{\pi} R \, \mathrm{Sin}^3\alpha \, d\alpha$$

Here, R is the intercepting length between the point on the meridional curve and axis of symmetry (x-axis) of the body and 'α' is the slope of normal. In Cartesian coordinates, 'h' can be expressed as

(1.4) $$h = \left(-\frac{3}{4}\right) \int_0^a \frac{yy''}{(1+y'^2)^2} dx,$$

where, x = a is maximum axial length and dashes represents derivative with respect to x.

The Transverse Flow

Let us consider an axially-symmetric body placed in a uniform stream 'U' along transverse axis (y-axis). The Stokes drag on this body is given to be [1].

(1.5) $$F_y = \left(\frac{1}{2}\right) \frac{\lambda (y_{\max})^2}{h_y},$$

where

$$\lambda = 6\pi\mu U$$

and

(1.6) $$h_y = \left(\frac{3}{16}\right) \int_{\alpha=0}^{\pi} (2R \, \mathrm{Sin}\,\alpha - R \, \mathrm{Sin}^3\alpha) \, d\alpha,$$

In Cartesian coordinates, h_y can be expressed as

(1.7) $$= \left(\frac{3}{8}\right) \int_0^a \left[\frac{yy''}{(1+y'^2)} - \frac{yy''}{(1+y'^2)^2}\right] dx.$$

For the details, the reader is referred to the paper [1]. Now, in the next section, the method for Stokes drag on axi-symmetric body placed in the uniform stream attacking at an angle 'ζ' with the axis of symmetry, is proposed.

2. The Method

Let us consider an axially symmetric body placed in uniform stream 'U' attacking an angle of attack 'ζ' with the axis of symmetry (x-axis) [see, Fig. 1]

Let us consider $\mathbf{e}_x, \mathbf{e}_y$ as unit vectors representing x and y directions and F_x, F_y are axial and transverse drags, then force vector F can be written as

(2.1) $$F = F_x \mathbf{e}_x + F_y \mathbf{e}_y,$$

where F_x and F_y (axial and transverse Stokes drags) : are defined in (1.1) and (1.5). Since the uniform stream U makes an angle 'ζ' with the x-axis (Fig.1) then its components U Cos ζ and U Sin ζ will be in x and y direction and in general we can have the axial and transverse drag as

(2.2) $$F_{x_1} = F_x \operatorname{Cos} \zeta,$$

and

(2.3) $$F_{y_1} = F_y \operatorname{Cos} \zeta,$$

Then force vector **F** can be written as

$$\mathbf{F} = F_{x_1} \mathbf{e}_x + F_{y_1} \mathbf{e}_y$$

(2.4) $$= F_x \operatorname{Cos} \zeta\, \mathbf{e}_x + F_y \operatorname{Sin} \zeta\, \mathbf{e}_y,$$

and its magnitude will be given by $F = |\mathbf{F}|$ which reduces to axial and transverse drag as $\zeta = 0$ and $\zeta = \pi/2$.

Now, in the next section, we use the result (2.4) to obtain force vector with magnitude for the axi-symmetric bodies.

3. Flow Past Spheroid Prolate Spheroid

Let us consider the prolate spheroid, generated by the rotation of ellipse

(3.1) $$x = a \operatorname{Cos} t, \quad y = b \operatorname{Sin} t, \quad 0 \le t \le \pi,$$

about the x-axis.

By using (2.4), together with (1.1) and (1.5), the expression for drag will be

(3.2) $$F = 16\pi \mu U a e^3 \left[\frac{\operatorname{Cos}^2 \varsigma}{\{-2e + (1+e^2)L\}^2} + \frac{4 \operatorname{Sin}^2 \varsigma}{\{2e + (3e^2 - 1)L\}^2} \right]^{\frac{1}{2}},$$

where, $L = \ln \dfrac{(1+e)}{(1-e)}$.

Oblate Spheroid

Let us consider the oblate spheroid, generated by the rotation of ellipse

(3.3) $x = a \operatorname{Cos} t, \quad y = b \operatorname{Sin} t, \quad 0 \le t \le \pi$
about x-axis

By using (2.4), together with (1.1) and (1.5), the expression for drag will be

$$F = 8\pi\mu U a e^3 \left[\frac{\operatorname{Cos}^2 \varsigma}{\left\{ e\sqrt{(1-e^2)} - (1-2e^2)\operatorname{Sin}^{-1} e \right\}^2} + \right.$$

(3.4)
$$\left. + \frac{4\operatorname{Sin}^2 \varsigma}{\left\{ -e\sqrt{(1-e^2)} + (1+2e^2)\operatorname{Sin}^{-1} e \right\}^2} \right]^{\frac{1}{2}}$$

4. Flow Past a Deformed Sphere

Consider the axially symmetric body defined by

(4.1)
$$r = a\left[1 + \varepsilon \left\{ d_0 + d_2 P_2(\mu) + \sum_{k=0}^{\infty} d_{2k+1} P_{2k+1}(\mu) \right\} \right], \quad \mu = \operatorname{Cos}\theta,$$

where (r, θ) are spherical polar coordinates and $P_k(\mu)$ is Legenedre function of first kind. For small parameter ε, this represents a deformed sphere. By using (2.4), together with (1.1) and (1.5), the expression for drag will be

(4.2)
$$F = \lambda a \left[1 + 2\varepsilon \left\{ d_0 + \frac{d_2}{10}(1 - 3\operatorname{Sin}^2 \varsigma) \right\} + 0(\varepsilon^2) \right]^{\frac{1}{2}}$$

5. Flow Past Cycloidal Body of Revolution

A. Let us take the inverted cycloid

(5.1) $x = a(t + \operatorname{Sin} t), \quad y = a(1 + \operatorname{Cos} t), \quad -\pi \le t \le \pi :,$

with vertex at (0, 2a), and revolve it about x-axis, the base, to generate the cycloidal body of revolution. By using (2.4), together with (1.1) and (1.5), the expression for drag is given to be

(5.2)
$$F = \frac{128}{15}\mu U a \, [36 - 11\operatorname{Cos}^2 \varsigma]^{\frac{1}{2}}.$$

B. Let us consider the body generated by the rotation about x-axis of the curve composed of arcs of two cycloidal parts represented parametrically by

$$\text{(5.3)}\qquad \left.\begin{array}{l} x = a(1+\text{Cos } t),\ y = a(t+\text{Sin } t),\ 0 \le t \le \pi; \\ x = -a(1+\text{Cos } t),\ y = a(t+\text{Sin } t),\ 0 \le t \le \pi; \end{array}\right\}$$

by using (2.4), together with (1.1) and (1.5), the required drag will be

$$\text{(5.4)}\qquad F = 96\pi^3 \mu \text{Ua} \left[\frac{\text{Cos}^2\varsigma}{(3\pi^2+16)^2} + \frac{4(1-\text{Cos}^2\varsigma)}{(9\pi^2+32)^2} \right]^{\frac{1}{2}}$$

6. Flow Past an Egg-Shaped Body

Let us consider an egg-shaped body in which right portion is in the shape of a half prolate spheroid given parametrically by

$$\text{(6.1)}\qquad \left.\begin{array}{c} x = a\ \text{Cos } t,\ \ y = b\ \text{Sin } t,\ 0 \le t \le \pi/2\,, \\ \text{and the left half portion is a hemisphere given by} \\ x = b\ \text{Cos } t,\ \ y = b\ \text{Sin } t,\ \pi/2 \le t \le \pi. \end{array}\right\}$$

By using (2.4) together with (1.1) and (1.5), the drag is given by

$$\text{(6.2)}\qquad F = 8\pi \mu \text{Ua} \sqrt{1-e^2} \left[\frac{\text{Cos}^2\varsigma}{\left\{ \frac{2}{3} + \frac{\sqrt{1-e^2}(-2e+(1+e^2)L)}{4e^3} \right\}^2} \times \frac{4(1-\text{Sin}^2\varsigma)}{\left\{ \frac{4}{3} + \frac{\sqrt{1-e^2}(2e+(3e^2-1)L)}{4e^3} \right\}^2} \right]^{\frac{1}{2}} + L$$

7. Flow Past Cassini Body of Revolution

Let us consider the cassini body obtained by revolving the curve

$$\text{(7.1)}\qquad y^2 = \frac{2}{3}(1+3x^2)^{\frac{1}{2}} - x^2 - \frac{1}{3}, \quad 0 \le x \le 1,$$

about the axis of symmetry (x-axis).

By using (2.4) together with (1.1) and (1.5), the expression for drag will be

$$\left.\begin{aligned} F &\approx [(0.8)^2 \operatorname{Cos}^2 \varsigma + (0.82)^2 \operatorname{Sin}^2 \varsigma]^{\frac{1}{2}} \lambda, \lambda = 6\pi \mu U \\ &\approx [0.6724 - 0.0324 \operatorname{Cos}^2 \varsigma]^{\frac{1}{2}} \lambda. \end{aligned}\right\}, \tag{7.1}$$

8. Flow Past Hypocycloidal Body of Revolution

Let us consider the body generated by rotating the curve

$$y^2 = -3x^2 + (1+8x^4)^{\frac{1}{2}}, \quad 0 \le x \le 1, \tag{8.1}$$

about axis of symmetry (x-axis).

Now, the expression for drag can be written by the use of (2.4), together with (1.1) and (1.5)

$$\left.\begin{aligned} F &\approx [(1.044)^2 \operatorname{Cos}^2 \varsigma + (1.32)^2 (1 - \operatorname{Cos}^2 \varsigma)]^{\frac{1}{2}} \lambda, \lambda = 6\pi \mu U \\ &\approx [1.7424 - 0.6525 \operatorname{Cos}^2 \varsigma]^{\frac{1}{2}} \lambda. \end{aligned}\right\}, \tag{8.2}$$

All the above results are found to be new for the axially symmetric bodies lies under the restricted class [1] and never existed in the literature

Acknowledgments

The present work has been done under the minor research project of UGC of the Govt. Post graduate College (Kumaun University, Nainitial), Pithoragarh (Uttaranchal). Author is thankful to the authorities of college to provide necessary facilities during the completion of the present work.

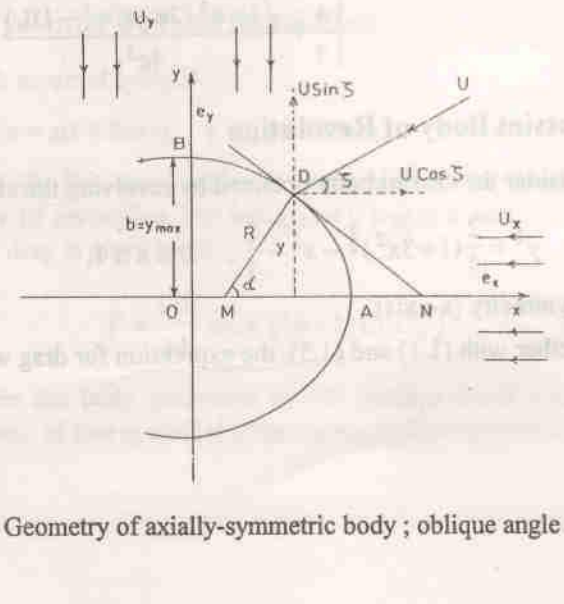

Fig. 1- Geometry of axially-symmetric body ; oblique angle of attack

REFERENCES

[1] Datta, S., and Srivastava, D. K., "*Stokes drag on axially symmetric bodies : a new approach*". Proc. (Math. Sci.) I.A. Sci. Vol. 109, No. 4 (Nov. 1999), 441–452.

[2] Happle, J. And Brenner, H. "*Low Reynolds Number Hydrodynamics*" Printice Hall. Englewood, 1965, Clifs N. J.

DEEPAK KUMAR SRIVASTAVA
Department of Mathematics
B.S.N.V. Post Graduate College,
(Lucknow University, Lucknow)
Lucknow (U.P)–226001. India.

E-mail : dksflow@hotmail.com