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# A Note On The Derivation of A Class of Bilateral Generating Function For The Konhauser Polynomials By Lie Group Theoretic Method

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**Abstract:** A new class of bilateral generating function for the Konhauser polynomials $Y_n^{\alpha}(x,k)$ [3] is obtained.

**Keywords:** Special Function, generating functions, Lie group

## 1. introduction:

Applying L. Weisner's group theoretic method [2] we obtained the following operator for $Y_n^{\alpha}(x, k)$ by giving suitable interpretation to $n$.

(1.1) $$R = yx\frac{\partial}{\partial x} + ky^2\frac{\partial}{\partial y} + (\alpha - x + 1)y$$

such that

(1.2) $$R[e^{ny} n!\, Y_n^{\alpha}(x;k)] = k(n+1)!\, e^{(n+1)y}\, Y_{n+1}^{\alpha}(x;k)$$

The extended form of the transformation group is

(1.3) $$\exp wR\,[f(x,t)] = (1-wkt)^{-(\alpha+1)/K} \exp[x\{1-(1-wkt)^{-1/k}\}] \times f(x(1-wkt)^{-1/K}, t(1-wkt)^{-1})$$

where $f(x, t)$ is an arbitrary differentiable function.

The object of the present paper is to derive a general class of bilateral generating functions by employing group theoretic method. Actually our results can be put in the form of a theorem as follows

**Theorem:** *If there exists a unilateral generating function of the form*

(1.4) $$G(x,t) = \sum_{n=0}^{\infty} n!\, a_n Y_n^{\alpha}(x,k)\, t^n$$

then the following class of bilateral generating functions will hold

$$(1.5) \qquad (1-kt)^{-(\alpha+1)/K} \exp[x\{1-(1-kt)^{-1/k}\}]\, G[x(1-kt)^{-1/k}\},\, ty(1-kt)^{-1}]$$
$$= \sum_{p=0}^{\infty} \sum_{n=0}^{p} a_n \frac{k^{(p-n)} p!}{(p-n)!} t^p y^n Y_p^{\alpha}(x,k)$$

Importance of the result (1.5) is that whenever one knows a generating function of the type (1.4) for a particular value of $a_n$ then the corresponding bilateral generating functions can at once be written from (1.5).

Thus, one can derive a large number of bilateral generating functions by setting different values to $a_n$.

**2. Proof of the theorem:**

Let

$$(2.1) \qquad G(x,t) = \sum_{n=0}^{\infty} n!\, a_n Y_n^{\alpha}(x,k)\, t^n$$

Replacing $t$ by $ty$, we have

$$G(x,t) = \sum_{n=0}^{\infty} n!\, a_n Y_n^{\alpha}(x,k)\, t^n y^n$$

we operate both sides by exp $wR$ and hence

$$(2.2) \qquad \exp wR\, [G(x, ty)] = \exp wR \sum_{n=0}^{\infty} n!\, a_n Y_n^{\alpha}(x,k)\, t^n y^n$$

Left hand side of (2.2) becomes

$$(1-wkt)^{-(\alpha+1)/K} \exp[x\{1-(1-wkt)^{-1/k}\}]\, f\,(x\{1-(1-wkt)^{-1/k},\, ty(1-wkt)^{-1})$$

On the other hand, right side of (2.2) reduces to

$$\sum_{p=0}^{\infty} \frac{w^p R^p}{p!} \sum_{n=0}^{\infty} n!\, a^n Y_p^{\alpha}(x,k)\, t^n y^n$$
$$= \sum_{p=0}^{\infty} \sum_{n=0}^{p} a_n \frac{w^p}{p!} k^p (n+p)!\, t^{(n+p)}\, y^n\, Y_{n+p}^{\alpha}(x,k)$$
$$= \sum_{p=0}^{\infty} \sum_{n=0}^{p} a_n \frac{w^{(p-n)}}{(p-n)!} p!\, k^{(p-n)}\, t^p\, y^n\, Y_p^{\alpha}\, Y_p^{\alpha}(x,k)$$

equating and substituting $w = 1$, we obtain (1.5)

**Application** : Setting $k = 1$, we have the following bilateral generating function for $L_n^{\alpha}(x)$.

If $G(x,t) = \sum_{n=0}^{\infty} a_n n!\, L_n^{\alpha}(x)\, t^n$

then we have

$$(1-t)^{-(\alpha+1)} \exp\left[\frac{xt}{1-t}\right] G[x(1-t)^{-1}, ty(1-t)^{-1}]$$

$$= \sum_{p=0}^{\infty}\sum_{n=0}^{p} a_n \frac{p!}{(p-n)!} t^p y^n L_p^{\alpha}(x)$$

This was derived by *Al Salam* [1]

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