The Nepali Math. Sc. Report
Vol. 30, No. 1 & 2, 2010

Additive Derivation on Involution *- Rings

TULASI PRASAD NEPAL
Central Department of Mathematics
Tribhuvan University
Kirtipur, Kathmandu, Nepal
Email: tulasi.nepal@yahoo.com

Abstract: In this paper author gives some results on additive derivation on involution * -rings.

Key words: Additive mapping, Jordan subring, Jordan ideal, torsion free.

AMS classification: 16W10, 17C50, 17B60

1. Introduction

A ring R is prime if $xRy = 0$, $x, y \in R$ implies $x = 0$ or $y = 0$. We say that R is 2 torsion free if in R, $2x = 0$ forces $x = 0$. An additive mapping $x \mapsto x^*$ satisfying $(xy)^* = y^* x^*$ and $(x^*)^* = x$ is an involution. A ring equipped with an involution is a *- ring. We say that a subset A of a ring R is a Jordan subring of R if A is an additive subgroup such that for x, y in A, $xoy = xy + yx$ must also in A. A subset A of a ring is called a Lie subring of R if A is an additive subgroup such that for x, y in A, [x,y] = xy-yx must also in A.

Let A be a Jordan subring of R. The additive subgroup $U \subset A$ is said to be a Jordan ideal of A if whenever $u \in U$ and $x \in A$ then $uox = ux + xu$ is in U. Let A be a Lie subring of R. The additive subgroup $U \subset A$ is said to be a Lie ideal of A if whenever $u \in U$ and $x \in A$ then [u,x] = $ux - xu$ is in U.

Combining (1) and (2), we have

$$2D(x)\, y*x*+2xD(y)\, x*+2xyD(x) = 2D(xyx)$$

$$D\,(xyx) = D(x)\, y*x*+xD(y)x*+xyD(x)$$

Proof of the theorem 2.3: Since $D\,(xyx) = D(x)y*x*+xD(y)x*+xyD(x)$ for all $x,y \in U$.

Linearizing on x

$$D\,((x + z)y(x + z)) = D(x+z)y*(x+z)*+(x+z)D(y)(x+z)*+(x+z)yD(x+z)$$

$$D(xyx+xyz+zyx+zyz) = (D(x)+D(z))y*(x*+z*)+(x+z)D(y)(x*+z*)+(x+z)y(D(x)+D(z))$$

$$D(xyx)+D(zyz)+D(xyz+zyx) = D(x)y*x* +D(x)y*z*+ D(z)y*x*+D(z)y*z*+xD(y)x*+ xD(y)z* +zD(y)x*+zD(y)z*+xyD(x)+xyD(z)+zyD(x)+zyD(z)$$

$$D\,(xyz+zyx) = D(x)\, y*z*+ xD(y)z*+xyD(z)+D(z)y*x*+zD(y)x*+zyD(x)$$

for all $x,y,z \in U$.

REFERENCES

[1] I.N. Herstein," *Rings with Involution*", University of Chicago Press, Chicago 1976.

[2] M. Breŝar and J. Vukman,"*On some additive mappings in rings with involution*", Aequationes Mathematical 38(1989), P 178-185.

□□□