The Nepali Math. Sc. Report
Vol. 26, No. 1 & 2, 2006.

Analytic solution for a system of KDV equations

MAHENDRA PANTHEE

Abstract: We consider a system of coupled Korteweg-de Vries equations and prove well-posedness results in a space of functions analytic in a strip. The typical class of functions we consider to obtain analytic solution is the Gevery class introduced by Foias and Temam in [6].

1. Introduction

In this work we consider the initial value problem (IVP)

$$(1.1)\qquad \begin{cases} u_t + u_{xxx} + 2\alpha u u_x + v v_x + (uv)_x = 0 \\ v_t + v_{xxx} + 2\beta v v_x + u u_x + (uv)_x = 0 \\ u(x,0) = u_0(x), v(x,0) = v_0(x) \end{cases}$$

where α, β are constants with $\alpha + \beta = 1$ and $x, t \in \mathbb{R}$ This is a system studied by Nutku and Oğuz in [16] and has a structure of the Korteweg-de Vries (*KdV*) equations coupled in the nonlinear terms. This system has a bi-Hamiltonian structure. If the constants are such that $\alpha = \pm\beta$, then the equations in the system (1.1) can be decoupled.

The main interest of this work is to find solutions $(u(x,t), v(x,t))$ of the IVP (1.1) which admit an extension as an analytic function to a complex strip $S_\sigma := \{x + iy : |y| < \sigma\}$, at least for small values of σ. Analytic Gevrey class introduced by Foias and Temam [6] is a suitable function space for our purpose.

In recent literature, many authors have devoted much effort to get analytic solutions to several evolution equations. An early work in this direction is due to Kato and Masuda [12]. They considered a large class of evolution equations and developed a general method to obtain spatial analyticity of the solution. In particular, the class considered in [12] contains the KdV equation. The more recent results in this field can be found in the work of Hayashi [10], Hayashi and Ozawa [11], de Bouard, Hayashi and Kato [3], Kato and Ozawa [13], Bona, Grujić and Kalish [1, 2], Grujić and Kalish [8, 9] and references there in.

Let us move to introduce some notations and define space of functions in which we will concentrate our work. For $\sigma > 0$ and $s \in \mathbb{R}$, the analytic Gevrey class $G^{\sigma,s}$ is defined as the subspace of $L^2(\mathbb{R})$ with norm.

$$(1.2)\qquad \|f\|^2_{G^{\sigma,s}} = \int_{\mathbb{R}} \langle\xi\rangle^{2s} e^{2\sigma\langle\xi\rangle} |\hat{f}(\xi)|^2\, d\xi,$$

where $\langle\,.\,\rangle = 1 + |\,.\,|$ and $\hat{f}$ denotes the Fourier transform of f defined by

This was partially supported by CAMGSD, IST through FCT/POCTI-POCI/FEDER and FCT/SFRH/BPD/22018/2005, Portugal.

(1.3) $$\hat{f}(\xi)=\frac{1}{\sqrt{(2\pi)}}\int_{\mathbb{R}} e^{-ix\xi} f(x)\,dx,$$

whose inverse transform is given by

(1.4) $$f(x)=\frac{1}{\sqrt{(2\pi)}}\int_{\mathbb{R}} e^{ix\xi}\hat{f}(\xi)\,d\xi.$$

If we define a Fourier multiplier operator A by

$$\widehat{Af}(\xi)=\langle\xi\rangle\hat{f}(\xi)$$

then Gevrey norm of order (σ, s) can be written in terms of the operator A as

$$\| f\|_{G^{\sigma,s}}=\| A^s e^{\sigma A} f\|_{L^2(\mathbb{R})}$$

Note that a function in the Gevrey class $G^{s,\sigma}$ is a restriction to the real axis of a function analytic on a symmetric strip of width 2σ. Hence, our interest is to prove well-posedness result for the IVP (1.1) for given data in $G^{\sigma,s}\times G^{\sigma,s}$ for appropriate s.

Before establishing well-posedness results in the analytic Gevrey class, we will prove the same in the usual Sobolev spaces $H^s\times H^s$. Recall that, H^s denotes the L^2-based Sobolev space of order s with norm

$$\| f\|_{H^s}^2=\int_{\mathbb{R}}\langle\xi\rangle^{2s}|\hat{f}(\xi)|^2\,d\xi,$$

We denote by $L_t^p(L_x^q)$, $(1<p<\infty)$ the Banach spaces $L^p(\mathbb{R}:L^q(\mathbb{R}))$ for variables t and x respectively. For $-1<b<1$, let $X_{s,b}$ denote the Hilbert space with the norm

$$\| f\|_{X_{s,b}}=\left(\int_{\mathbb{R}^2}(1+|\tau-\xi^3|)^{2b}(1+|\xi|)^{2s}|\hat{f}(\xi,\tau)|^2\,d\xi\,d\tau\right)^{1/2},$$

where $\hat{f}(\xi,\tau)$ is the Fourier transform of f in both x and t variables. This is the space introduced by Bourgain [4] in the KdV context to obtain well-posedness results for low regularity data.

Let us recall some properties of the space $X_{s,b}$ regarding the regularity. First, observe that for $f\in X_{s,b}$, one has,

$$\| f\|_{X_{s,b}}=\|(1+D_t)^b U(t) f\|_{L_t^2(H_x^s)},$$

where $U(t)=e^{-t\partial_x^3}$ is the unitary group associated with the linear *KdV* flow. If $b>1/2$, the previous remark and the Sobolev lemma imply,

$$X_{s,b}\subset C(\mathbb{R};H_x^s(\mathbb{R})).$$

We use C to denote various constants whose exact values are immaterial. Also, we use the notation $A\lesssim B$ if there exists a constant $C>0$ such that $A<CB$, $A\gtrsim B$ if there exists a constant $C>0$ such that $A>CB$ and $A\sim B$ if $A\lesssim B$ and $A\gtrsim B$.

Now we state the local existence result for given data in the usual Sobolev space $H^s(\mathbb{R})\times H^s(\mathbb{R})$.

Theorem 1.1. *For any* $(u_0,v_0)\in H^s(\mathbb{R})\times H^s(\mathbb{R})$, $s>-3/4$ *and* $b\in(1/2,1)$, *there exist* $T=T(\| u_0\|_{H^s},\| v_0\|_{H^s})$ *and a unique solution of (1.1) in the time interval* $[-T,T]$ *satisfying*

(1.5) $$u, v \in C([-T, T]; H^s(\mathbb{R})),$$

(1.6) $$u, v \in X_{s,b} \subseteq L^p_{x,loc}(\mathbb{R}; L^2_t(\mathbb{R})), \quad for \ 1 \le p \le \infty,$$

(1.7) $$(u^2)_x, (v^2)_x \in X_{s,b-1}$$

and

(1.8) $$u_t, v_t \in X_{s-3,b-1}.$$

Moreover, given $T' \in (0,T)$, the map $(u_0, v_0) \mapsto (u(t), v(t))$ is smooth from $H^s(\mathbb{R}) \times H^s(\mathbb{R})$ to $C([-T', T']; H^s(\mathbb{R})) \times C([-T', T']; H^s(\mathbb{R}))$.

Note that $\int (u^2 + v^2)\, dx$ is conserved by the flow of (1.1). Using this conserved quantity we can obtain an a *priori* estimate in $L^2(\mathbb{R}) \times L^2(\mathbb{R})$ which leads to the following global well-posedness result.

Theorem 1.2. *The unique local solution to the initial value problem (1.1) obtained in Theorem 1.1 can be extended globally in time for given data in $H^s(\mathbb{R}) \times H^s(\mathbb{R})$, whenever $s \ge 0$.*

Remark 1.3. Using I-method and almost conserved quantity introduced in the KdV *context by Colliander et al in* [5], *the global well-posedness result of the above theorem can be improved for $s > -3/10$. There is similar work in this direction by the author in collaboration with Linares in* [17]. *As our interest here is to obtain analytic solutions, we do not proceed in this direction.*

Before stating the main result of this work, let us introduce the function space $X^{\sigma,s,b}$, which is analogue of Bourgain's space $X_{s,b}$ introduced earlier.

For $\sigma > 0$ and $s \in \mathbb{R}$, $b \in [-1, 1]$ define $X^{\sigma,s,b}$ with the norm

$$\| f \|^2_{H^{\sigma,s,b}} = \iint \langle \tau - \xi^3 \rangle \langle \xi \rangle^{2s} e^{2\sigma \langle \xi \rangle} | \hat{f}(\xi, \tau) |^2 \, d\xi \, d\tau.$$

If we define the operator Λ^ρ, for $\rho \in \mathbb{R}$ by

$$\widehat{\Lambda^\rho f}(\xi, \tau) = \langle \tau \rangle^\sigma \hat{f}(\xi, \tau).$$

then we have

$$\| Uf \|_{X^{\sigma,s,b}} = \| A^s e^{\sigma A} \Lambda^b f \|_{L^2(\mathbb{R}^2)}.$$

Let us record that $C([0, T]; G^{\sigma,s})$ denotes the space of continuous functions defined on the interval $[0, T]$ that take values in $G^{\sigma,s}$. If we equip $C([0, T]; G^{\sigma,s})$ with the norm

$$\sup_{0 \le t \le T} \| f(.,.) \|_{G^{\sigma,s}}$$

then it becomes a Banach space.

For $b > 1/2$, using Sobolev embedding we have

$$\sup_{0 \le t \le T} \| f(.,.) \|_{G^{\sigma,s}} \le c \| u \|_{X^{\sigma,s,b}}.$$

Therefore the space $X^{\sigma,s,b}$ is embedded in $C([0, T]; G^{\sigma,s})$ whenever $b > 1/2$. Now we are in position to state the main result of this work which reads as follows.

Theorem 1.4. Let $s \geq 0$ *and* $\sigma > 0$ *then for any* $(u_0, v_0) \in G^{\sigma,s} \times G^{\sigma,s}$, *there exists a time* $T > 0$ *such that the IVP* (1.1) *is well- posed in the space* $C([0,T]; G^{\sigma,s}) \times C([0,T]; G^{\sigma,s})$.

2. Well-posedness result in usual Sovolev space

In this section we will prove well-posedness results in the usual Sobolev spaces. The idea of the proof is similar to the one employed for the Gear and Grimshaw system in the author's previous work in collaboration with Linares in [17]. For the sake of completeness, we just give sketch of the proof.

Proof of Theorem 1.1. Using Duhamel's principle, we study the following system of integral equations equivalent to the system (1.1),

$$(2.1)\qquad \begin{cases} u(t) = U(t)u_0 - \int_0^t U(t-t')\, F(u,v,u_x,v_x)(t')\,dt', \\ v(t) = U(t)v_0 - \int_0^t U(t-t')\, G(u,v,u_x,v_x)(t')\,dt', \end{cases}$$

where $U(t) = e^{(-t\partial_x^3)}$ is the unitary group that describes the linear KdV flow and F and G are respective nonlinearities.

To find a local solution to the IVP (1.1) we can replace the system (2.1) with the following system

$$(2.2)\qquad \begin{cases} u(t) = \psi_1(t)U(t)u_0 - \psi_1(t) \int_0^t U(t-t')\,\psi_T(t')\, F(u,v,u_x,v_x)(t')\,dt', \\ v(t) = \psi_1(t)U(t)v_0 - \psi_1(t) \int_0^t U(t-t')\,\psi_T(t')\, G(u,v,u_x,v_x)(t')\,dt', \end{cases}$$

where $\psi \in C_0^\infty(\mathrm{IR})$, $0 \leq \psi(t) \leq 1$ is a smooth function such that

$$(2.3)\qquad \psi(t) = \begin{cases} 1, & |t| \leq 1, \\ 0, & |t| \leq 2, \end{cases}$$

and $\psi_T(t) = \psi(\frac{t}{T})$, $0 < T \leq 1$.

Now, we consider the following function space where we seek a solution to the IVP (1.1). For given $(u_0, u_0) \in H^s(\mathrm{IR}) \times H^s(\mathrm{IR})$ and $b > 1/2$, let us define,

$$\mathcal{H}_{MN} := \{(u,v) \in X_{s,b} \times X_{s,b} : \| u \|_{X_{s,b}} \leq M,\ \| v \|_{X_{s,b}} \leq N\},$$

where $M = 2C_0 \| u_0 \|_{H^s}$ and $N = 2C_0 \| v_0 \|_{H^s}$ Then $\mathcal{H}_{MN}$ is a complete metric space with norm,

$$|||(u,v)|||_{\mathcal{H}_{MN}} := \| u \|_{X_{s,b}} + \| v \|_{X_{s,b}}.$$

Without loss of generality, we may assume that $M > 1$ and $N > 1$. For $(u,v) \in \mathcal{H}_{MN}$, let us define the maps,

$$(2.4)\qquad \begin{cases} \Phi_{u_0}[u,v] = \psi_1(t)U(t)u_0 - \psi_1(t) \int_0^t U(t-t')\,\psi_T(t') F(u,v,u_x,v_x)(t')\,dt' \\ \Psi_{v_0}[u,v] = \psi_1(t)U(t)v_0 - \psi_1(t) \int_0^t U(t-t')\,\psi_T(t') G(u,v,u_x,v_x)(t')\,dt', \end{cases}$$

We prove that $\Phi \times \Psi$ maps $\mathcal{H}_{MN}$ into $\mathcal{H}_{MN}$ and is a contraction. To achieve this goal we use the following estimates

(2.5) $$\|\psi_1 U(t)u_0\|_{X_{s,b}} \le C\|u_0\|_{H^s},$$

(2.6) $$\|\psi_T \int_0^t U(t-t')f(t')dt'\|_{X_{s,b}} \le CT^{1-b+b'}\|f\|_{X_{s,b'}},\ b>\frac{1}{2},\ b-1<b'<0$$

and

(2.7) $$\|\partial_x(uv)\|_{X_{s,b'}} \le c\|u\|_{X_{s,b}}\|v\|_{X_{s,b}},\ s>-\frac{3}{4},\ \frac{1}{2}<b<\frac{3}{4},\ b-1<b'<-\frac{1}{4},$$

Proof of estimates (2.5) and (2.6) is given in [14] and [7] and that of (2.7) is given in [15]. Now using estimates (2.5)-(2.7) we obtain

(2.8) $$\begin{cases} \|\Phi[u,v]\|_{X_{s,b}} \le C_0\|u_0\|_{H^s} + C_1T^\theta\{\|u\|^2_{X_{s,b}} + \|v\|^2_{X_{s,b}} + \|u\|_{X_{s,b}}\|v\|_{X_{s,b}}\} \\ \|\Psi[u,v]\|_{X_{s,b}} \le C_0\|v_0\|_{H^s} + C_2T^\theta\{\|u\|^2_{X_{s,b}} + \|v\|^2_{X_{s,b}} + \|u\|_{X_{s,b}}\|v\|_{X_{s,b}}\}, \end{cases}$$

where $\theta = 1-b+b'$.

As $(u,v)\in \mathcal{H}_{MN}$, with our choice of M and N we get from (2.8),

(2.9) $$\begin{cases} \|\Phi[u,v]\|_{X_{s,b}} \le \frac{M}{2} + C_1T^\theta\{M^2+N^2+MN\} \\ \|\Psi[u,v]\|_{X_{s,b}} \le \frac{N}{2} + C_2T^\theta\{M^2+N^2+MN\}. \end{cases}$$

If we choose T such that,

$$T^\theta \le (2\max\{C_1,C_2\}(M+N)^2)^{-1}$$

then the estimate (2.9) yields,

$$\|\Phi[u,v]\|_{X_{s,b}} \le M \text{ and } \|\Psi[u,v]\|_{X_{s,b}} \le N.$$

Therefore,

$$(\Phi[u,v],\Psi[u,v]) \in \mathcal{H}_{MN}.$$

In an analogous manner we can show that $\Phi\times\Psi : (u,v)\mapsto(\Phi[u,v],\Psi[u,v])$ is a contraction.

Therefore the map $\Phi\times\Psi$ is a contraction map in the ball $\mathcal{H}_{MN}$. Hence, there exists a unique fixed point (u,v) that solves the IVP (1.1) for $T\le\delta$. The remainder of the proof follows a standard argument. □

3. Well-posednesss results in the analytic class

Now we proceed to establish the estimates that are fundamental in the proof of the main result of this work.

3.1.Linear estimates.

Lemma 3.1. *Let $s\in\mathbb{R}$, $\sigma>0$, $u_0\in G^{\sigma,s}$, $b>1/2$ and $b-1<b'<0$. Then there exists a constant C such that*

(3.10) $$\|\psi(t)U(t)u_0\|_{X^{\sigma,s,b}} \le \|u_0\|_{G^{\sigma,s}},$$

(3.11) $$\|\psi_T\int_0^t U(t-t')f(t')dt'\|_{X^{\sigma,s,b}} \le CT^{1-b+b'}\|f\|_{X^{\sigma,s,b'}}$$

Proof: For $\sigma=0$ the estimates in (3.10) and (3.11) turn to be estimates (2.5) and (2.6) respectively. For $\sigma>0$, we just need to replace u_0 by $e^{\sigma A}u_0$ and f by $e^{\sigma A}f$ and so the proof follows in analogous manner. □

3.2. Bilinear estimate

Lemma 3.2. *Let $u,v \in X^{\sigma,s,b}$, $s \geq 0$, $\sigma > 0$, $\frac{1}{2} < b < \frac{3}{4}$. If $b-1 < b' < -\frac{1}{4}$, then there exists a constant C depending only on s, b and b′ such that*

$$\| \partial_x(uv) \|_{X^{\sigma,s,b'}} \leq C \| u \|_{X^{\sigma,s,b}} \| v \|_{X^{\sigma,s,b}}. \tag{3.12}$$

Proof: We give proof of (3.12) for $s = 0$, the general case $s > 0$ follows from it. So, our interest here is to prove

$$\| \partial_x(uv) \|_{X^{\sigma,0,b'}} \leq C \| u \|_{X^{\sigma,0,b}} \| v \|_{X^{\sigma,0,b}}. \tag{3.13}$$

Let us define

$$f(\xi,\tau) = \langle \tau - \xi^3 \rangle^b e^{\sigma\langle \xi \rangle} \hat{u}(\xi,\tau),$$
$$g(\xi,\tau) = \langle \tau - \xi^3 \rangle^b e^{\sigma\langle \xi \rangle} \hat{v}(\xi,\tau).$$

So that, $\| u \|_{X^{\sigma,0,b}} = \| f \|_{L^2_\xi L^2_\tau}$ and $\| v \|_{X^{\sigma,0,b}} = \| g \|_{L^2_\xi L^2_\tau}$. Also,

$$\| \partial_x(uv) \|_{X^{\sigma,0,b'}} = \| \langle \tau - \xi^3 \rangle^{b'} e^{\sigma\langle \xi \rangle} \xi (\hat{u} * \hat{v})(\xi,\tau) \|_{L^2_\xi L^2_\tau}$$

$$= \left\| \langle \tau - \xi^3 \rangle^{b'} e^{\sigma\langle \xi \rangle} \xi \iint \hat{u}(\xi_1,\tau_1)\, \hat{v}(\xi - \xi_1, \tau - \tau_1)\, d\xi_1 d\tau_1 \right\|_{L^2_\xi L^2_\tau} \tag{3.14}$$

$$= \left\| \frac{e^{\sigma\langle \xi \rangle} \xi}{\langle \tau - \xi^3 \rangle^{-b'}} \iint \frac{f(\xi_1,\tau_1) e^{-\sigma\langle \xi_1 \rangle}}{\langle \tau_1 - \xi_1^3 \rangle^b} \frac{g(\xi - \xi_1, \tau - \tau_1) e^{-\sigma\langle \xi - \xi_1 \rangle}}{\langle \tau - \tau_1 - (\xi - \xi_1)^3 \rangle^b} d\xi_1 d\tau_1 \right\|_{L^2_\xi L^2_\tau}$$

Now, the estimate (3.13) can be written in terms of f and g as

$$\left\| \frac{\xi e^{\sigma\langle \xi \rangle}}{\langle \tau - \xi^3 \rangle^{-b'}} \iint \frac{e^{-\sigma\langle \xi_1 \rangle} f(\xi_1,\tau_1) e^{-\sigma\langle \xi - \xi_1 \rangle} g(\xi - \xi_1, \tau - \tau_1)}{\langle \tau_1 - \xi_1^3 \rangle^b \langle \tau - \tau_1 - (\xi - \xi_1)^3 \rangle^b} d\xi_1 d\tau_1 \right\|_{L^2_\xi L^2_\tau} \tag{3.15}$$
$$\leq C \| f \|_{L^2_\xi L^2_\tau} \| g \|_{L^2_\xi L^2_\tau}.$$

Using Cauchy-Schwarz inequality and Fubiani's theorem, the LHS of (3.15) can be estimated as

$$\left\| \frac{\xi e^{\sigma\langle \xi \rangle}}{\langle \tau - \xi^3 \rangle^{-b'}} \iint \frac{e^{-\sigma\langle \xi_1 \rangle} f(\xi_1,\tau_1) e^{-\sigma\langle \xi - \xi_1 \rangle} g(\xi - \xi_1, \tau - \tau_1)}{\langle \tau_1 - \xi_1^3 \rangle^b \langle \tau - \tau_1 - (\xi - \xi_1)^3 \rangle^b} d\xi_1 d\tau_1 \right\|_{L^2_\xi L^2_\tau}$$

$$\leq \left\| \frac{\xi e^{\sigma\langle \xi \rangle}}{\langle \tau - \xi^3 \rangle^{-b'}} \iint \frac{e^{-\sigma\langle \xi_1 \rangle} e^{-\sigma\langle \xi - \xi_1 \rangle} d\xi_1 d\tau_1}{\langle \tau_1 - \xi_1^3 \rangle^b \langle \tau - \tau_1 - (\xi - \xi_1)^3 \rangle^b} \right\|_{L^\infty_\xi L^\infty_\tau} \| f \|_{L^2_\xi L^2_\tau} \| g \|_{L^2_\xi L^2_\tau}. \tag{3.16}$$

So, to obtain the desired estimate (3.15) and there by (3.13), we need to show

$$(3.17)\qquad \left\| \frac{\xi e^{\sigma\langle\xi\rangle}}{\langle\tau-\xi^3\rangle^{-b'}} \iint \frac{e^{-\sigma\langle\xi_1\rangle} e^{-\sigma\langle\xi-\xi_1\rangle}\, d\xi_1 d\tau_1}{\langle\tau_1-\xi_1^3\rangle^b \langle\tau-\tau_1-(\xi-\xi_1)^3\rangle^b} \right\|_{L^\infty_\xi L^\infty_\tau} < C.$$

Note that, by triangle inequality we have $|\xi| \le |\xi_1| + |\xi-\xi_1|$. So, $e^{\sigma\langle\xi\rangle} \le e^{\sigma\langle\xi_1\rangle} e^{\sigma\langle\xi-\xi_1\rangle}$ and the estimate (3.17) will be proved if we can show

$$(3.18)\qquad \left\| \frac{\xi}{\langle\tau-\xi^3\rangle^{-b'}} \iint \frac{d\xi_1 d\tau_1}{\langle\tau_1-\xi_1^3\rangle^b \langle\tau-\tau_1-(\xi-\xi_1)^3\rangle^b} \right\|_{L^\infty_\xi L^\infty_\tau} < C.$$

The expression in (3.18) is exactly the same term appeared in the proof of the usual bilinear estimate related to the KdV equation in [15]. So, the rest of the proof follows the same lines in [15]. This completes the proof of the lemma. □

3.3. Proof of the main result. Now we will use the linear and bi-linear estimates derived above to prove the main result of this work.

Proof of Theorem 1.4. The idea of proof is the similar to that of Theorem 1.1 presented earlier. We write the IVP (1.1) in its equivalent integral form as in (2.2). For given $(u_0,v_0) \in G^{\sigma,s}(\mathrm{IR}) \times G^{\sigma,s}(\mathrm{IR})$ and $b > 1/2$, let us define $M = 2C_0 \| u_0 \|_{G^{\sigma,s}}$ and $N = 2C_0 \| v_0 \|_{G^{\sigma,s}}$. Now define a ball

$$\mathcal{B}_{MN} := \{(u,v) \in X^{\sigma,s,b} \times X^{\sigma,s,b} : \| u \|_{X^{\sigma,s,b}} \le M, \| v \|_{X^{\sigma,s,b}} \le N\}.$$

Then $\mathcal{B}_{MN}$ is a complete metric space with norm,

$$|\| (u,v) \||_{\mathcal{B}_{MN}} := \| u \|_{X^{\sigma,s,b}} + \| v \|_{X^{\sigma,s,b}}$$

In this case also, without loss of generality, we may assume that $M > 1$ and $N > 1$. For $(u,v) \in \mathcal{B}_{MN}$, let us define the maps $\Phi \times \Psi$ as in (2.5). We will show that $\Phi \times \Psi$ is a contraction map in the ball $\mathcal{B}_{MN}$.

First, let us move to show that $\Phi \times \Psi$ maps the ball $\mathcal{B}_{MN}$ into itself. Using estimates (3.10) – (3.12) we obtain as in (2.9), for $\theta = 1 - b + b'$,

$$(3.19)\qquad \begin{cases} \|\Phi\|_{X^{\sigma,s,b}} \le \frac{M}{2} + C_1 T^\theta \{M^2 + N^2 + MN\} \\ \|\Psi\|_{X^{\sigma,s,b}} \le \frac{N}{2} + C_2 T^\theta \{M^2 + N^2 + MN\}. \end{cases}$$

Now, choosing T such that,

$$T^\theta \le (2\max\{C_1, C_2\}(M+N)^2)^{-1}$$

we obtain from (3.19),

$$\|\Phi\|_{X^{\sigma,s,b}} \le M \text{ and } \|\Psi\|_{X^{\sigma,s,b}} \le N.$$

Therefore, $\Phi \times \Psi$ maps $\mathcal{B}_{MN}$ into $\mathcal{B}_{MN}$. One can easily prove that $\Phi \times \Psi$ is a contraction map in an analogous manner, so we skip it.

Hence, the map $\Phi \times \Psi$ has a unique fixed point (u,v) which solves the IVP (1.1) for $T \le \delta$ in the ball $\mathcal{B}_{MN}$. The rest of the proof follows a standard argument so we omit the details. This completes the proof of the theorem. □

REFERENCES

[1] Bona, J. L., Grujić, Z., Kalish H., ; *Algebraic lower bounds for the uniform radius of spatial analyticity for the generalized KdV equation*, Ann. I.H. Poincaré, **AN22** (2005) 783–797.

[2] Bona, J. L., Grujić, Z., Kalish H., ; *Global solutions for the derivative Schrödinger equation in a class of functions analytic in a strip*, J. Differential Equations, In press.

[3] de Bouard, A., Hayashi N., Kato, K., *Gevrey regularizing effect for the (generalized) Korteweg-de Vries equation and nonlinear Schrödinger equations*, Ann. Inst. H. Poincaré Anal. Non Lineairé, **6** (1995) 673–715.

[4] Bourgain, J.; *Fourier transform restriction phenomena for certain lattice subsets and applications to nonlinear evolution equations*, Geom. Funct. Anal., **3** (1993) Part I ; 107–156; Paart II: 209–262.

[5] Colliander J., Keel, M., Staffilani, G., Takalka, H., Tao, T., ; *Global well-posedness for KdV in Sobolev spaces of negative index*, Electron, J. Diff. Eqn. 2001, No. 26, 7 (pp), electronic.

[6] Foias, C., Temam, R., ; *Gevrey class regularity for the solutions of the Navier-Stokes equations*, J. Funct. Anal. **87** (1989), 359–369.

[7] Ginibre, J., Tsutsumi, Y., Velo, G., ; *On the Cauchy problem for the Zakharov system*, J. Funct. Anal. **151** (1997), 384–436.

[8] Grujić, Z., Kalish H., ; *Local well-posedness of the generalized Korteweg-de Vries equation in spaces of analytic functions*, Differential and Integral Equations, **15** (2002), 1325-1334.

[9] Grujić, Z., Kalish H., ; *The Derivative Nonlinear Schrödinger Equation in Analytic Class*, J. Nonlinear Math. Phy, **10** (2003), 62–71.

[10]Hayashi, N., *Global existence of small analytic solutions to nonlinear Schrödinger equations*, Duke Math. J., **60** (1990), 717–727.

[11]Hayashi, N., Ozawa, T., ; *Remarks on nonlinear Schrödinger equations in one space dimension*, Differential Integral Equations, **7** (1994), 453–461.

[12]Kato, T., Masuda, K., ; *Nonlinear evolution equations and analyticity I*, Ann. Inst. H. . Poincaré Anal. Non Linéaire, **3** (1986), 455–467.

[13]Kato, K., Ogawa, T., ; *Analyticity and smoothing effect for the Korteweg-de Vries equation with a single point singularity*, Math. Ann., **316** (2000), 577–608.

[14]Keing, C. E., Ponce, G., Vega, L., ; *On the Cauchy problem for the Korteweg-de Vries equation in Sobolev spaces of negative indices*, Duke Math. J. **71** (1993), 1–20.

[15] Keing, C. E., Ponce, G., Vega, L., ; *A bilinear estimate with application to the KdV equation*, J. Amer. Math Soc., **9** (1996), 573–603.

[16]Nutku, Y., Oğuz, Ö., *Bi-Hamiltonian structure of a pair of coupled KdV equations*, Nuovo Cimento, **11** (1990), 1381–1383.

[17]Linares, F., Panthee, M., ; *On the Cauchy problem for a coupled system of KdV equations*, Comm. On Pure and Appl. **33** (2004), 417–431.

MAHENDRA PANTHEE

CENTRAL DEPARTMENT OF MATHEMATICS,
TRIBHUVAN UNIVERSITY, KIRTIPUR, KATHMANDU, NEPAL.

DEPARTMENT OF MATHEMATICS, CAMGSD,
INSTITUTO SUPERIOR TÉCNICO,
AV. VOVISCO PAIS, 1049-001, LISBON, PORTUGAL.
E-mail address: mpanthee@math.ist.Utl.pt