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Application in Bio-Mathematics of Hypergeometrid Type

B.R. SHARMA

Introduction

In recent research work, Ronghe [4] has determined the equations of Atmospheric pressure and half life period, making use of Fox's H-functions. We shall now here establish certain results involving generalized H-functions which lead in Bio-mathematics. This paper concludes with some interesting special cases for G-functions of the main results established herein. Meijer's G-functions of two variables introduced by Agarwal (I) was extended by the introduction of H-function of two variables by Mittal and Gupta (3). They have given the following notations to define the H-function of two variables as:

$$(1)\qquad H^{0,n:(m_1,n_1);(m_2,n_2)}_{p,q,:(r,s);(k,1)}\left[\begin{array}{c|c} x & (a_1;A_j,\alpha_j)_{1p} \\ & (c_j,C_j)_{1r},(e_j,E_j)_{1k} \\ y & (b_j;B_j,\beta_j)_{1q} \\ & (d_1,D_j)_{1s}(f_j,F_j)_{11} \end{array}\right]$$

$$= -\frac{1}{4\pi}\int_{L_1}\int_{L_2}\theta(\xi,\eta)g(\xi)h(\eta)x^{\xi}y^{\eta}d\xi d\eta,$$

where,

$$g(\xi,\eta)=\frac{\prod_{j=1}^{n}\Gamma(1-a_j+A_j\xi+\alpha_j\eta)}{\prod_{j=m}^{n}\Gamma(1-b_j+B_j\xi+\beta_j\eta)\prod_{j=n+1}^{n}\Gamma(a_j-A_j\xi-\alpha_j\eta)}$$

$$g(\xi)=\frac{\prod_{j=1}^{m_1}\Gamma(d_j-D_j\xi)\prod_{j=1}^{n_1}\Gamma(1-e_j+C_j\xi)}{\prod_{j=m_1+1}^{s}\Gamma(1-d_j+D_j\xi)\prod_{j=n_1+1}^{k}\Gamma(e_j-C_j\xi)},$$

$$h(\eta)=\frac{\prod_{j=1}^{m_2}\Gamma(f_j-F_j\eta)\prod_{j=1}^{n_2}(1-e_j+E_j\eta)}{\prod_{j=m_2+1}^{m_1}\Gamma(1-f_j+F_j\eta)\prod_{j=n_2+1}^{k}\Gamma(e_j-E_j\eta)} \text{ valid for}$$

(i) $/x/</, /y/</, 0\le n\le p, 0\le m_1\le g, 0\le n_1\le s, 0\le m_2\le k, 0\le n_2\le 1.$

(ii) All $A'S, B'S, C'S, D'S, E'S, F'S$, α's and $\beta' S$ are positive quantities.

(iii) The H-function 1.1(1) converges provided

(a) $U=-\sum_{j=n+1}^{p}A_j-\sum_{j=1}^{q}B_j+\sum_{j=1}^{m_1}D_j-\sum_{j=m_1+1}^{S}D_j+\sum_{j=1}^{n_1}C_j-\sum_{j=n_1+1}^{r}C_j>0$

(b) $V=-\sum_{j=n+1}^{p}d_j-\sum_{j=1}^{q}\beta_j+\sum_{j=1}^{m_2}F_j-\sum_{j=m_2+1}^{1}F_j+\sum_{j=1}^{n_2}E_j-\sum_{j=n_2+1}^{k}E_j>0$

(c) $/\ arg x\ /<\frac{1}{2}\pi U,$

(d) $/\ arg y\ /<\frac{1}{2}\pi V$

1.2. In this section we shall determine the Equation of Population Growth involving H-function of two variables.

Suppose that $P(t)$ is the Population size at time t and δP the growth in the Population corresponding to time δt, then we have

(1) $\delta P \ \alpha\ P\delta t$, which gives the differential equation

(2) $DP/dt=\lambda P$,

where λ is proportional constant.

Integration of 1.2(2) yields.

$\int\frac{dp}{dt}=\lambda p+k_1$, where k_1 is a constant of integration and therefore

(3) $$\int\frac{\Gamma(P)}{\Gamma(P+1)}dP=\lambda\frac{\Gamma(t+1)}{\Gamma(t)}+k_1$$

We can now determine the value of constant k_1 at the initial conditions $t=0$. $P=P_0$. When time increases, the population also increases. If we make $P\to P+P_1\xi$ and $t\to t+t_1\xi$, then 1.2(3) gives

(4) $$\int\frac{\Gamma(P+P_1\xi)}{\Gamma(1+P+P_1\xi)}dP=\lambda\frac{\Gamma(1+t+t_1\xi)}{\Gamma(t+t_1\xi)}+k_1$$

Similarly, we can deduce.

(5) $$\int\frac{\Gamma(Q+Q_1\eta)}{\Gamma(1+Q+Q_1\eta)}dQ=\lambda_1\frac{\Gamma(1+t^1+t_1^1\eta)}{\Gamma(t^1+t_1^1\eta)}+k_2$$

Combine 1.2(4) and 1.2(5) and multiply both sides of it by $\frac{1}{(2\pi i)^2}\Phi(\xi,\eta)g(\xi)h(\eta)$, further, integrate with respect to ξ and η along the direction of double contours L_1 and L_2 and use 1.1(1) to obtain the following equation of population growth involving H-function of two variables :

$$(6)\quad \iint H^{0,n:(m_1+1;n_1+1);(m_2+1,n_2+1)}_{p,q:(r+1,s+1);(k+1,l+1)}\left[\begin{matrix}x\\y\end{matrix}\left|\begin{matrix}(a_p;A_p,\alpha_p)\\(1-P,P_1);(c_r,C_r);(1-Q,Q_1),(c_k,E_r)\\(b_q;B_q,\beta_q)\\(d_s,D_s),(-P,P_1;(f_1,F_1),(-Q,Q_{1_r})\end{matrix}\right.\right]dPdQ$$

$$= \mu_1 H^{0,n:(m_1+1,n_1+1);(m_2+1,n_2+1)}_{p,q:(r+1,s+1);(k+1,1+1)}\left[\begin{matrix}x\\y\end{matrix}\left|\begin{matrix}(a_p;A_p,\alpha_p)\\(-t,t_1);(c_r,C_r);(-t^1,t_1^1),(e_k,E_k)\\(b_q;B_q,\beta_q)\\(d_s,D_s),(1-t_1);(f_1,f_1),(1-t^1,t_1^1)\end{matrix}\right.\right]$$

$$+\mu_2 H^{0,n:(m_1+1,n_1+1);(m_2,n_2)}_{p,q:(r+1,s+1);(k,1)}\left[\begin{matrix}x\\y\end{matrix}\left|\begin{matrix}(a_p;A_p,\alpha_p)\\(-t,t_1);(c_r,C_r);(e_k,E_k)\\(b_q;B_q,\beta_q)\\(d_s,D_s),(1-t,t_1);(f_1,F_1)\end{matrix}\right.\right]$$

$$+\mu_3 H^{0,n:(m_1,n_1);(m_2+1,n_2+1)}_{p,q:(r,s);(k+1,1+1)}\left[\begin{matrix}x\\y\end{matrix}\left|\begin{matrix}(a_p;A_p,\alpha_p)\\(c_r,C_r);(-t^1,t_1),(e_k,E_k)\\(b_q;B_q,\beta_q)\\(b_q,D_s);(f_1,F_1),(1-t^1,t_1^1)\end{matrix}\right.\right]$$

$$+\mu_4 H^{0,n:(m_1,n_1);(m_2+n_2)}_{p,q:(r,s);(k+1)}\left[\begin{matrix}x\\y\end{matrix}\left|\begin{matrix}(a_p;A_p,\alpha_p)\\(c_r,C_r);(e_k,E_k)\\(b_q;B_q,\beta_q)\\(d_{s_2}D_s);(f_1,F_1)\end{matrix}\right.\right]$$

where

(i) $\mu_1(-\lambda\lambda_1)$, $\mu_2(=\lambda k_2)$, $\mu_3(=\lambda_1 k_1)$ and $\mu_4(=k_1k_2)$ are all constants

(ii) $P_1>0$, $Q_1>0$, $t_1>0$ and $t_1^1>0$.

1.3. Special Cases:

If we put $A_j=\alpha_j=1$, (j=1,2, ... , p), $B_j=\beta_j=1$(j=1,2, ... , q), $C_j=1$ (j=1,2, ... , r), $D_j=1$ (j=1,2, ... , s), $E_j=1$ (j=1,2, ... , k),

$F_j = 1$ $(j=1,2,\ \ldots, I)$, $P_1 = 1$, $Q_1 = 1$, $t_1 = 1$ and $t_1^1 = 1$. in 1.2(6), then we have

$$(1)\quad \iint G\,{}^{n,n_1+1,n_2+1,m_1+1,m_2+1}_{p\,(r+1,k+1)\,q,(s+1\,;\,1+1)}\left[\begin{matrix}x\\y\end{matrix}\left|\begin{matrix}(a_p)\\(1-P),(c_r);(1-Q),(e_k)\\(b_q)\\(d_s),(-P)\,;(f_1),(-Q)\end{matrix}\right.\right]dPdQ$$

$$= \mu_1 G\,{}^{n,n_1+1,n_2+1,m_1+1,m_2+1}_{p\,(r+1;k+1),q,(s+1\,;\,1+1)}\left[\begin{matrix}x\\y\end{matrix}\left|\begin{matrix}(a_p)\\(-t),(c_r);(-t^1),(e_k)\\(b_q)\\(d_s),(1-t)\,;(f_1),(1-t^1)\end{matrix}\right.\right]$$

$$+\mu_2 G\,{}^{n,n_1+1,n_2,m_1+1,m_2}_{p,(r+1;k),q,(s+1\,;\,1)}\left[\begin{matrix}x\\y\end{matrix}\left|\begin{matrix}(a_p)\\(-t),(c_r);(e_k)\\(b_q)\\(d_s),(1-t)\,;(f_1),\end{matrix}\right.\right]$$

$$+\mu_3 G\,{}^{n,n_1,n_2+1,m_1,m_2+1}_{p,(r;k+1),q,(s:1+1)}\left[\begin{matrix}x\\y\end{matrix}\left|\begin{matrix}(a_p)\\(c_r);(-t^1),(e_k)\\(b_q)\\(d_s),(f_1),(1-t^1)\end{matrix}\right.\right]$$

$$+\mu_4 G\,{}^{n,n_1,n_2,m_1,m_2}_{p,(r;k),q,(s:1)}\left[\begin{matrix}x\\y\end{matrix}\left|\begin{matrix}(a_p)\\(c_r)\,;(e_k)\\(b_q)\\(d_s)\,;(f^1)\end{matrix}\right.\right]$$

where μ_1, μ_2, μ_3 and μ_4 are all constants and the G-functions of two variables in 1.3(1) are valid for

$/x < 1$, $/y/ < 1$,

$0 \le n \le p$, $0 \le n_1 \le r$, $0 \le n_2 \le k$, $0 \le m_1 \le s$, $0 \le m_2 \le 1$, $0 \le q$. and for convergence, we have

$$p+q+s+r < 2\,(m_1+n_1+n)$$

$$p+q+1+k < 2\,(m_2+n_2+n)$$

$$/argx/ < \pi\ [m_1+n_1+n-(p+q+s+r)/2]$$

$$/argx/ < \pi\ [m_2+n_2+n-(p+q+1+k)/2]$$

In particular, if we take $p = 0$ and $q = 0$ in 1.3(1) we have.

$$(2)\quad \iint G^{m_1+1,n_1+1}_{r+1,s+1}\left[x\left|\begin{matrix}(1-p),(C_r)\\(d_s),(-P)\end{matrix}\right.\right]\times G^{m_2+1,n_2+1}_{k+1,1,+1}\left[y\left|\begin{matrix}(1-Q),(e_k)\\(f_1),(-Q)\end{matrix}\right.\right]dPdQ$$

$$= \mu_1\, G^{m_1+1,n_2+1}_{r+1,s+1}\left[x\left|\begin{matrix}(-t),(e_r)\\(d_s),(1-t)\end{matrix}\right.\right]\times G^{m_2+1,n_2+1}_{k+1,1,+1}\left[v\left|\begin{matrix}(-t^1),(e_k)\\(f_1),(1-t^1)\end{matrix}\right.\right]$$

$$+ \mu_2\, G^{m_1+1,n_1+1}_{r+1,s+1}\left[x\left|\begin{matrix}(-t),(e_r)\\(d_s),(1-t)\end{matrix}\right.\right]\times G^{m_2,n_2}_{k,1}\left[y\left|\begin{matrix}(e_k)\\(f_1)\end{matrix}\right.\right]$$

$$+ \mu_3\, G^{m_1,n_1}_{r,s}\left[x\left|\begin{matrix}(e_r)\\(d_s)\end{matrix}\right.\right]\times G^{m_2+1,n_2+1}_{k+1,1,+1}\left[y\left|\begin{matrix}(-t^1),(e_k)\\(f_1),(1-t^1)\end{matrix}\right.\right]$$

$$+ \mu_3\, G^{m_1,n_1}_{r,s}\left[x\left|\begin{matrix}(e_r)\\(d_s)\end{matrix}\right.\right]\times G^{m_2,n_2}_{k,1}\left[y\left|\begin{matrix}(e_k)\\(f_1)\end{matrix}\right.\right].$$

where μ_1, μ_2, μ_3 and μ_4 are all constants and the G-functions in 1.3 (2) are valid for $/x/<1$, $/y/<1$,

$$0 \le m_1 \le s,\ 0 \le m_1 \le r,\ 0 \le m_2 \le 1,\ 0 \le n_2 \le k$$
$$r+s < 2\,(m_1+n_1)$$
$$k+1 < 2\,(m_2+n_2)$$
$$/argx/ < \pi\ [m_1+n_1-(r+s)/2],$$
$$/argy/ < \pi\ [m_2+n_2-(k+1)/2]$$

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B.R. SHARMA
Dept. Of Mathematics
Campus Chief
Mahendra Multiple Campus (T.U)
Dang., Nepal.