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Baro-diffusion and Thermal–diffusion in a Binary Mixture Near A Stagnation Point–A Numerical Study

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Abstract: The effects of pressure gradient and temperature gradient on separation of a binary mixture of incompressible viscous fluids have been discussed when one of the components of the fluid mixture is present in a small quantity. The flow has been discussed when a stream of such a mixture impinges on an impervious wall at right angles and flows along this wall in all radial directions. Equations of motion and energy together with the equation for species conservation have been solved numerically by Runga-Kutta shooting technique. It has been found that there is no separation effect when pressure gradient and temperature gradient are ignored. The effects of the pressure gradient as well as the temperature gradient are to separate the two components of the mixture in such a manner that the heavier and more abundant component gets deposited near the wall.

1. Introduction

Consider a mixture of two components of fluids the composition of one of which is described by the concentration c_1 defined as the ratio of mass of that component to the total mass of the fluid in a given volume element. In the flow of such a mixture the diffusion of individual species takes place by three mechanisms namely concentration gradient, pressure gradient and temperature gradient. The diffusion flux $\vec{i}$ is given by Landau and Lifshitz [5] as :

$$\vec{i} = -\rho D[\nabla c_1 + k_p \nabla p + k_T \nabla T] \tag{1}$$

where ρ is the density of the binary mixture, D is the diffusion coefficient, k_p is baro-diffusion ratio, $k_p D$ is the baro-diffusion coefficient, p is the pressure, k_T is thermal diffusion ratio, $k_T D$ is the thermal-diffusion coefficient and T is the temperature. The first term in the right hand side of equation (1) represents the ordinary diffusion whose contribution to the mass flux depends in a complicated way on the concentration gradients of the substances present in the binary mixture. The second one representing the pressure diffusion term indicates that there may be a net movement of the i^{th} species in the mixture if there is a pressure gradient imposed on the system. The last one representing the thermal diffusion term describes the tendency for species to diffuse under the influence of a temperature gradient. The effects of the last two terms are quite small, but devices can be arranged to produce very steep pressure gradients and temperature gradients so that separations of mixtures may be effected.

Much interest has been attached to the separation processes wherein one of the components is present in extremely small proportion in normal occurrence of the mixture. Separation of isotopes from their naturally occurring mixture is one such example. It is well known that because of their small relative mass difference the isotopes of heavier molecules offer greatest practical challenge to isolate the rarer component. Sharma [7] has discussed the problem of baro-diffusion in a binary mixture of viscous incompressible fluids when an infinite disk rotates with a constant angular velocity and there is a suction of the mixture at the disk. Srivastava [10] has discussed the baro-diffusion in a binary mixture confined between two disks when one of the disks is rotating and the other is at rest. Sharma and Gogoi [9] have discussed the effect of curvature of a curved annulus on separation of a binary mixture. Shrivastava [11] has discussed the baro-diffusion in a binary mixture in an axi-symmetric stagnation-point flow also. All these problems have been discussed under isothermal conditions. To investigate the effect of the temperature gradient in addition to the pressure gradient on separation of species of a binary mixture of thermally conducting incompressible fluids we have considered in this paper the flow discussed by Srivastava [11] when a stream of such a mixture impinges on a stationary impervious wall perpendicular to the stream and flows away in all radial directions. The equations of motion, energy and also the equation for conservation of species have been reduced to ordinary differential equations and numerical solutions of these ordinary non-linear differential equations have been obtained for various values of Schmidt number, baro-diffusion number, thermal-diffusion number and Prandtl number. Thus in this paper we have discussed the diffusion of the rarer component of the binary mixture under all the three effects namely the concentration gradient, the pressure gradient and the temperature gradient. Thus, the analysis provides a more accurate picture of the separation of the binary mixture of incompressible viscous fluids in the problem than the usual analysis under isothermal conditions.

2. Mass Transfer Equation

We consider here the case when one of the components of the binary mixture of incompressible fluids is present in a small quantity, hence the density and the viscosity of the mixture are independent of the distribution of the components. The flow problem of the binary mixture is identical to that of a single fluid but the velocity is to be understood as the mass average velocity $\vec{v} = (\rho_1 \vec{v}_1 + \rho_2 \vec{v}_2)/\rho$ and the density $\rho = \rho_1 + \rho_2$ where the subscripts 1 and 2 denote the rarer and more abundant components respectively. The equations of motion and the equation of continuity in the steady case are :

(2) $$\rho(\vec{v}\,.\nabla)\vec{v} = -\nabla_p + \mu\nabla^2\vec{v},$$

(3) $$\nabla.\vec{v} = 0,$$

where μ is the coefficient of viscosity of the binary mixture. The equation governing the temperature is given by

(4) $$\rho c_p(\vec{v}\,.\nabla)T = k\nabla^2 T + \phi_d,$$

where c_p is the specific heat at constant pressure, k is the thermal conductivity of the fluid and ϕ_d is the viscous dissipation function. The additional equation for the species concentration is given by

(5) $$\rho(\vec{v}\,.\nabla)c_1 = -\nabla.\vec{i}$$

Substituting $\vec{i}$ from equation (1) in (5); we get the following equation for c_1

(6) $$\rho(\vec{v}\,.\nabla)c_1 = \rho D[\nabla^2 c_1 + \nabla.(k_p\nabla_p) + \nabla.(k_T\nabla T)].$$

The explicit expression for the baro-diffusion ratio has been given by Landau and Lifshitz (5) as:

(7) $$k_p = (m_2 - m_1)[(c_1/m_1) + (c_2/m_2)\,c_1c_2/p_\infty,$$

where p_∞ denotes the pressure in the working medium, m_1 and m_2 are the masses of two kinds of particles and c_2 is the concentration of the second component of the binary mixture given by

(8) $$c_1 + c_2 = 1$$

since $c_1 = \rho_1/\rho$ and $c_2 = \rho_2/\rho$. Assuming c_1 to be small so that its square is negligible, we get the following expression for k_p :

(9) $$k_p = (m_2 - m_1)\,c_1/(m_2\,p_\infty).$$

The expression for k_T has been suggested by Hurle and Jakeman [4] as :

(10) $$k_T = c_1\,s_T(1 - c_1),$$

where s_T is Soret coefficient Neglecting the square of c_1 in this case also, we can write the expression for the thermal diffusion ratio k_T as:

(11) $$k_T = c_1 s_T.$$

3. Boundary Conditions on c_1

The boundary conditions on c_1 are different in different cases. At the solid surface of a body, insoluble in the fluid, the mass flux of the rarer component of the mixture normal to the solid surface is zero. This boundary condition can be written mathematically as :

(12) $\rho v_n c_1 - D[(\partial c_1/\partial n) + k_p(\partial p/\partial n) + k_T(\partial T/\partial n)] = 0$ at the surface,

where v_n is the fluid velocity normal to the surface and $\partial/\partial n$ denotes derivative normal to the surface. The first part represents the convective flux and the second part in the parenthesis denotes the diffusion flux. If, however, there is diffusion from a body that dissolves in the fluid, the equilibrium is rapidly established at the surface of the body and that the concentration in the fluid adjoining the body is, therefore, the saturation constant c_0. The boundary condition at such a surface is, therefore.

(13) $$c_1 = c_0.$$

4. Formulation of the Problem

In this section we discuss the flow, heat transfer and diffusion of a binary mixture of incompressible viscous fluids when a stream of such a mixture impinges on an impervious insulated wall $z = 0$ and flows away in all radial directions. We consider the temperature of the rarer component of the binary mixture, far away from the surface of the wall, to be constant. We take here cylindrical polar co-ordinates with stagnation point as the origin and the flow direction as negative z-axis. We denote the radial and axial component of the velocity in the frictionless flow region by U and W respectively whereas those in viscous flow region are denoted by $u = u\ (r,z)$ and $w = w\ (r,\ z)$. The boundary conditions on velocity and temperature fields are

(14) $u = 0\ , w = 0\ , (\partial T\ /\ \partial z\ ) = 0$ at $z = 0$ and $u \to \mathrm{U},\ T \to T_\infty$ as $z \to \infty$.

For frictionless case we have [See Schlichting and Gerston [8],

(15) $$\mathrm{U} = ar,\ W = -2az,\ P_0 = P + (\rho a^2/2)(r^2 + 4z^2),$$

where a is a constant and P_0 is the total pressure at the stagnation point. We take the following form for u, w and p in the viscous region :

(16) $$u = (av)^{1/2}\ \xi\,\phi'(\eta),\ w = -2(av)^{1/2}\ \phi(\eta),$$

(17) $$p - p_0 = \rho av P(\xi, \eta),$$

(18) $$T - T_\infty = (\mu c_p/k)\theta(\xi, \eta),$$

where $\eta = (a/v)^{1/2} z,\ \xi = (a/v)^{1/2} r,\ v = \mu/\rho$ and T_∞ is the temperature at a large distance from the surface of the wall.

Substituting expressions for u and w from (16), p from (17), T from (18), k_p from (9) and k_T from (11), the equation of motion (2) becomes

(19) $$\phi''' + 2\phi\phi'' - \phi'^2 + 1 = 0,$$

and also the expression for $P(\xi\ ,\eta)$ becomes

$$P(\xi,\eta) = -\frac{\xi^2}{2} - 2(\phi^2 + \phi'), \tag{20}$$

the equation of energy (4) reduces to

$$\Pr\left(\xi\phi'\frac{\partial\theta}{\partial\xi} - 2\phi\frac{\partial\theta}{\partial\eta}\right) = \left(\frac{\partial}{\xi\partial\xi}\left(\xi\frac{\partial\theta}{\partial\xi}\right) + \frac{\partial^2\theta}{\partial\eta^2}\right) + \Pr\left(12\phi'^2 + \xi^2\phi''^2\right) \tag{21}$$

and the diffusion equation (6) becomes

$$\begin{aligned} Sm\left(\xi\phi'\frac{\partial c_1}{\partial\xi}\right) - 2Sm\left(\phi\frac{\partial c_1}{\partial\eta}\right) &= \frac{\partial}{\xi\partial\xi}\left(\xi\frac{\partial c_1}{\partial\xi}\right) + \\ &+ \frac{\partial^2 c_1}{\partial\eta^2} + \beta\left(\frac{\partial}{\xi\partial\xi}\left(\xi c_1\frac{\partial P}{\partial\xi}\right) + \frac{\partial}{\partial\eta}\left(c_1\frac{\partial P}{\partial\eta}\right)\right) + \\ &+ \tau\left(\frac{\partial}{\xi\partial\xi}\left(\xi c_1\frac{\partial\theta}{\partial\xi}\right)\right) + \frac{\partial}{\partial\eta}\left(c_1\frac{\partial\theta}{\partial\xi}\right), \end{aligned} \tag{22}$$

where $\Pr = \mu c_p/k$ is the Prandtl number, $Sm = (v/D)$ is the Schmidt, β is the baro-diffusion number given by

$$\beta = \frac{m_2 - m_1}{m_2\, p_\infty} \cdot \frac{\rho a^2}{2}$$

and τ is the thermal diffusion number given by

$$\tau = \frac{\mu c_p}{k} \cdot S_T$$

The boundary conditions (14) on velocity and temperature become

$$\phi = 0,\ \phi' = 0,\ \theta' = 0 \text{ at } \eta = 0; \qquad \phi' \to 1,\ \theta \to 1 \text{ as } \eta \to \infty \tag{23}$$

and the boundary conditions (12) and (13) on c_1 become

$$\frac{\partial c_1}{\partial\eta} + \beta c_1\frac{\partial P}{\partial\eta} + \tau c_1\frac{\partial\theta}{\partial\eta} = 0 \text{ at } \eta = 0; \quad \text{and} \quad c_1 \to c_0 \quad \text{as } \eta \to \infty \tag{24}$$

5. Solution of Equations

To get solution of partial differential equations (21) and (22) we, at first, convert them to ordinary differential equations. For this we assume θ and c_1 in the forms:

$$\theta(\xi',\eta) = \theta_0 + \xi^2\theta_2 \tag{26}$$

and

$$c_1(\xi,\eta) = c_0 f(\xi,\eta) = c_0\left(f_0(\eta) + \xi^2 f_2(\eta)\right). \tag{20}$$

Putting these values of θ and c_1 in equations (21) and (22) and equating the coefficient of ξ^0 and ξ^2 separately from both sides, we get.

$$\theta_0'' + 4\theta_2 + 2\,\mathrm{Pr}\left(\phi\theta_0' + 6\phi'^2\right) = 0, \tag{27}$$

$$\theta_2'' - \mathrm{Pr}\left(2\phi'\theta_2 - 2\phi\theta_2' - \phi''^2\right) = 0, \tag{28}$$

$$\begin{aligned} Sm\left(-2\phi f_0'\right) = f_0'' + 4f_2 - 2\beta\left(2f_0 + 2\phi'^2 f_0 + 2\phi\phi'' f_0 + 2\phi\phi' f_0' + \phi''' f_0 + \phi'' f_1'\right) \\ + \tau\left(4f_0\theta_2 + f_0\theta_2'' + f_0'\theta_2'\right) \end{aligned} \tag{29}$$

$$\begin{aligned} 2Sm\left(\phi f_2 - \phi f_2'\right) = 2\beta\left(2f_2 + 2\phi'^2 f_2 + 2\phi\phi'' f_2 + 2\phi\phi' f_2' + \phi''' f_2 + \phi'' f_2'\right) \\ + \tau\left(8f_2\theta_2 + f_0\theta_2'' + f_0'\theta_2' + f_2\theta_0'' + f_2'\theta_2'\right) \end{aligned} \tag{30}$$

The boundary conditions (23) and (24) in terms of θ_0, θ_2, f_0 and become

$$\left.\begin{aligned} \theta_0' = 0, \theta_0(\infty) = 0 \\ \theta_2' = 0, \theta_2(\infty) = 0 \end{aligned}\right\} \tag{31}$$

and

$$f_0' - 2\beta\,\phi'' f_0 = 0 \ \text{ at } \ \eta = 0\,; \text{ and } \ f_0 \to 1 \ \text{ as } \ \eta \to \infty, \tag{32}$$

$$f_2' - 2\beta\,\phi'' f_2 = 0 \ \text{ at } \ \eta = 0\,; \text{ and } \ f_2 \to 0 \ \text{ as } \ \eta \to \infty \tag{33}$$

It is not possible to get analytical expressions for ϕ and hence for θ_0, θ_2, f_0 and f_2. Equation (19) was first solved numerically by Homann [3], and later by Frossling [2] under the boundary conditions (23). Here we have solved equations (19), (27)–(30) under boundary conditions (23), (31)–(33) numerically by using Runga-Kutta shooting technique [See Conte and Boor [1] Robert and Shlpman [6].

6. Discussion

For $\beta = 0$ and $\tau = 0$ the function $f(\xi, \eta)$ becomes 1 throughout the fluid, which shows that there is no separation effect in the binary mixture when pressure gradient and temperature gradient are ignored. This confirms the results of Sarma [7], Srivastava [11] and Sharma and Gogol [9]. The value of $f_2(\eta)$ is found to be zero for all values of η hence from (26) we can conclude that the concentration of the rarer component is independent of the distance from z axis. Taking Schmidt number Sm = 1, Prandti number Pr = 1 and Thermal diffusion number $\tau = 0$, the function $f(\eta)$ has been plotted for baro-diffusion number $\beta = 0.025, 0.050, 0.075$ and 0.100 in Fig. 1. The graph reveals that the value of f is always less than 1 near the upper edge of the boundary layer and its values at the wall are 0.777254, 0.603558, 0.467569 and 0.360866 respectively for the above mentioned values of β. This means that the concentration of the rarer component is much less near the wall than that maintained at a large distance from it. But, $c_1 + c_2 = 1$ which shows that the heavier component gets deposited more near the wall. This again confirms the results of Srivastava [11].

By taking $Sm = Pr = 1$ and $\beta = 0.025$, $f(\eta)$ has been plotted in Fig. 2 for τ = 0.000, 0.025 and 0.050. In this figure we see that the curves for above-mentioned values of τ intersect at $\eta = 0.2$. This indicates that the effect of the thermal diffusion number is to decrease the concentration of the rarer component from the upper edge of the boundary layer to the points corresponding to $\eta = 0.2$ and to decrease it beyond these points to the surface of the wall. The points corresponding to $\eta = 0.2$ are some special points, since concentration at these points remain unaffected by the thermal diffusion number.

In Fig. 3 we have plotted $f(\eta)$ for $Pr = 0.7$, 1.0 and 2.0 by taking $Sm = 1$ and $\beta = \tau = 0.025$. Fig. 4 represents the graph of $f(\eta)$ for Sm = 0.50, 0.75, 1.00, 1.25 and for fix values of $\beta = \tau = 0.025$ and $Pr = 1$. These graphs show that by reducing the values of Pr and/or Sm the rate of change in concentration of the rarer component of the binary mixture can be enhanced. Hence, we can conclude, from above analysis, that the effects of the pressure gradient and the temperature gradient are to collect the heavier component of the binary fluid mixture near the surface of the wall, i.e., to effect the separation of the species in the binary mixture. The rate of separation can be enhanced by reducing the values of Prandtl number and Schmidt number.

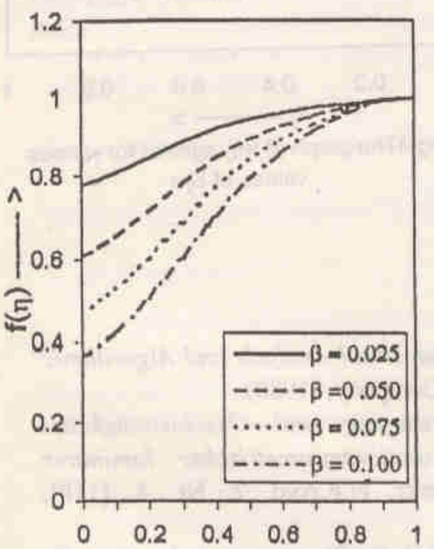

Fig.1The graph of f(η) against for various values of β

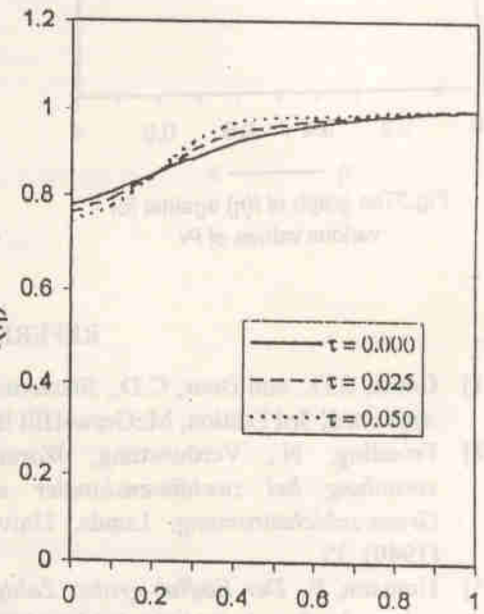

Fig.2The graph of f(η) against for various values of τ

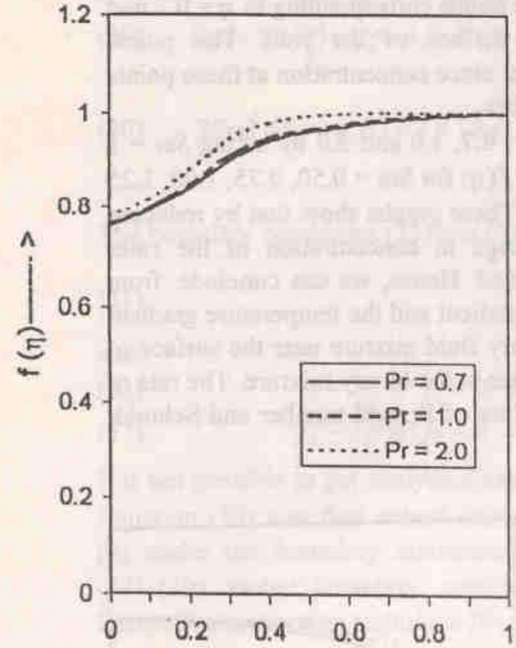

Fig.3The graph of f(η) against for various values of Pr

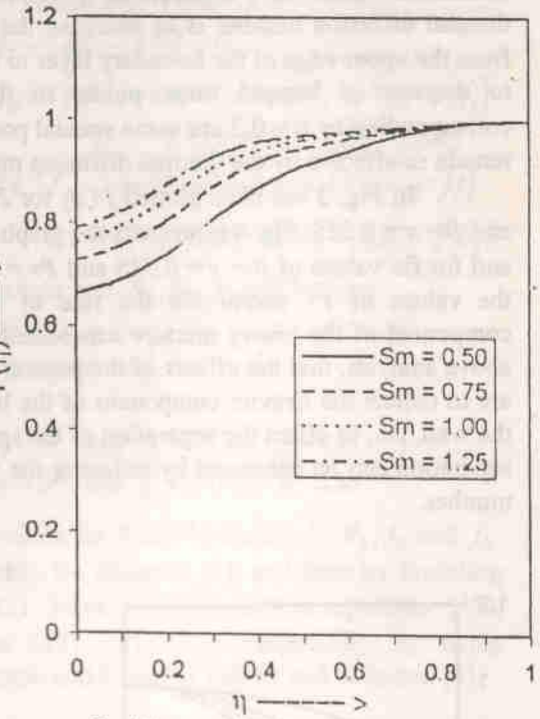

Fig.4The graph of f(η) against for various values of Sm

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