The Nepali Math. Sc. Report
Vol. 20 No. 1&2, 2002

Blood Flow Through Diseased Artery

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Abstract: Localized narrowing in any artery, commonly referred to as a stenosis, is a frequent result of arterial disease. It is caused mainly due to the intravascular atherosclerotic plaques, which develop at the arterial wall and protrude into the lumen of the vessels. Such contractions disturb normal blood flow through artery and there is considerable evidence that hydrodynamic factors such as wall shear stress, pressure distribution, separation etc. can play a significant role in the development and progression of this disease. Hence considerable attention has been given to the theoretical and experimental studies of stenotic region flow under different conditions. [Young {1968), Caro et al 1971), Deschpande et al (1977), Nerm (1974), Fox & Hugo (1966), Roadbard (1970). Rodkiewiez (1974), Steinman (2000), Gijsen F.I.H (1999).

Many authors have investigated analytically the flow characterisation of blood in artery with mild and non-critical stenosis.

In the above mentioned studies only the effect of single stenosis was considered and the tube has been taken to be of uniform cross-section. However, it may be noted that many of the blood vessels either converge or diverge slowly along their lengths. Also it is possible to have multiple stenosis in a series along the length of the tube.

In view of this, in this paper, the flow through tubes of non-uniform cross-section and with multiple stenosis is investigated. Solution for very mild stenosis have been obtained and the effects of heights of stenosis, number of stenosis etc. on the resistance to the flow λ and wall shear distribution γ have been studied.

Analysis

Let us consider the flow of blood through an artery having mild multiple stenosis in its lumen. We assume that the artery has two stenosis in series along the length and is such that the first stenosis is in a portion of uniform cross-section and the other one in a portion where the tube radius varies axially i.e. R=R*(z). Thus assuming that the two stenosis have developed symmetrically and are very mild, the geometry of the tube can be defined as:

$$\text{(1)}\quad R(z) = \begin{cases} R_0 : 0 \le z \le d_1 \text{ and } d_1 + L_1 \le z \le B_1 \\ \{R(z) : B_1 \le z \le d_2 \text{ and } d_2 + L_2 \le z \le B \\ \{R_0 - (\delta_1/2)\,\{1 + \text{Cos}\,(2\pi/L_1)\,(z - d_1 - L_1/2)\} : d_1 \le z \le d_1 + L_1 \\ \{R(Z) - (\delta_2/2)\,\{1 + \text{Cos}\,(2\pi/L_2)\,(z - d_2 - L_2/2)\} : d_2 \le z \le d_2 + L_2 \end{cases}$$

Where $R(z)$ is the tube radius at any cross-section, R_0 is the constant radius of the first portion, $R^*(z)$ is the tube radius in the second portion, δ_1 and δ_2 are the amplitude of the two stenoses and L_1 and L_2 are their lengths such that :

(2) $$\delta_1 \leq \min(R_0, R\text{ out}) \leq L_1$$

The basic equation governing the flow in a tube with mild constriction is,

(3) $$\therefore \quad dp/dz = (\mu/r)\, \partial/\partial r\, (\partial w/\partial r)$$

Where w is the axial velocity, p is the pressure, μ is the coeff. of viscosity. The boundary conditions are provided by the no–slip condition at the boundary of the tube and by the axial symmetry of the tube i.e.,

(4) $$\delta w/\partial r = 0 \quad \text{at } r = 0$$
$$w = 0 \quad \text{at } r = R(z)$$

The axial velocity w on solving eqn. (3) along with the conditions (4) is obtained as

(5) $$\omega = \{(-1/4\mu)\, \delta p/\delta z\}\, (R^2(z) - r^2)$$

The volumetric flow rate is defined as :

$$Q = \int_0^{R(z)} 2\pi r\, w\, dr$$

Which on using eqn. (5) is given by following expression :

(6) $$dp/dz = -8\mu Q / \pi R(z)$$

Integrating (6) alongwith the condition

$$\therefore p = p_1 \text{ at } z = 0$$

and $$p = p_0 \text{ at } z = B$$

We get

(7) $$p_1 - p_0 = (8\mu Q/\pi) \int_0^B dz/R^4(z)$$

Which gives the resistance to the flow λ as,

(8) $$\lambda = (p_1 - p_0)/Q = (8\mu/\pi) \int_0^B dz/R^4(z)$$

The shearing stress, τ_w, on the wall of the tube is given by,

(9) $$\tau = \{(1 - d\,R/dz)^2 / (1 + d\,R/dz)^2\}\, [4Q\mu/\pi R^3(Z)]$$

Let λ_n and τ_n be the resistance to the flow and the shearing stress at the wall for the flow of blood in a tube of uniform cross-section of radius R_0 and with no stenosis.

Then

(10) $$\lambda_n = 8\,\mu\; B/\pi R_0^4,$$

(11) $$\tau_n = 4\,\mu\; Q/\pi R_0^3$$

Thus from equations (8) -(10) and (9)-(11) we have,

(12) $$\bar{\lambda} = \lambda / \lambda_n = R_0^4 / B \int_0^B dz / R^4(Z)$$

(13) $$\bar{\tau} = \tau / \tau_n = \{R_0 / R(z)\}^3 [(1 - dR/dz)^2 / (1 + dR/dz)^2]$$

Discussion

The total resistance to the flow $\bar{\lambda}$ and the shear stress acting on the wall $\bar{\tau}$ are given by equations (12) and (13). To see explicitly the effects of the various parameters on the resistance to the flow and the wall shear the following function has been assumed for the tube radius in the second portion,

$$R^*(z) = [e^{K(Z-B_1)}]^2$$

Where K is the wall exponent parameter.

Numerical calculations have been done using the following values of the parameters:

$$\bar{B}_1 = 0.4$$
$$\bar{B}_2 = 1 \qquad K = -0.1, 0.0, +0.1$$
$$\bar{L}_1 = 0.04$$
$$\bar{L}_2 = 0.06$$
$$\bar{\delta}_1 = 0.0\text{–}0.2$$
$$\bar{\delta}_2 = 0.0\text{–}0.2$$

The resistances to the flow and the wall shear stresses are plotted in figures and the following effects have been observed :

(1) $\bar{\lambda}$ increases with the heights and the numbers of the stenosis.

(2) $\bar{\lambda}$ is more in a convergent tube compared to its value for a straight tube whereas far a divergent tube it decreases.

From the figures it can be seen that the sheer stress acting on the wall of the second region, τ_2 increases with the heights of the stenosis. For a convergent tube τ_2 is more compared to its value for a straight tube. However τ_2 decrease for a divergent tube compared to a straight tube.

Same behaviour is observed for the shear stress acting on the wall of the first region τ_1 These observations have been found to be in conformity with the experimental findings of Talukdar etc. al (Ref. 12).

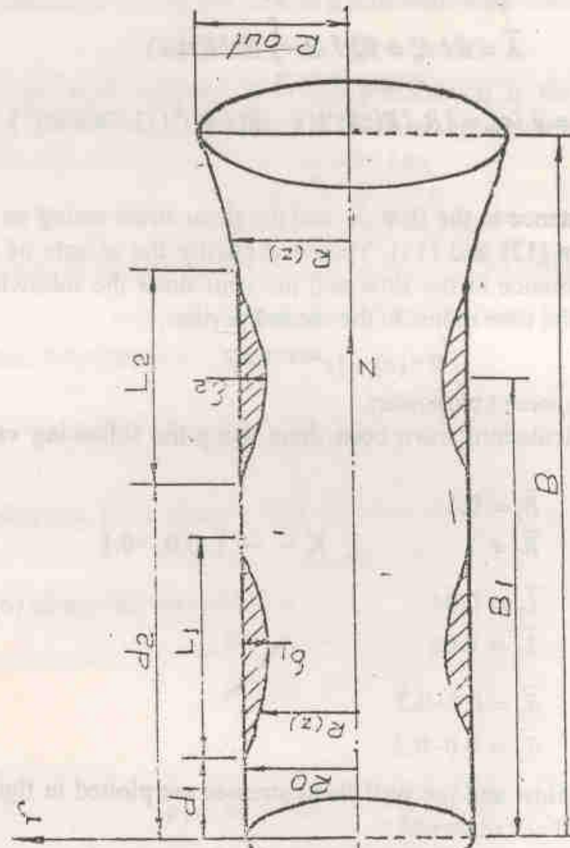

Fig 1.1 Geometry of the tube with multiple stenosis.

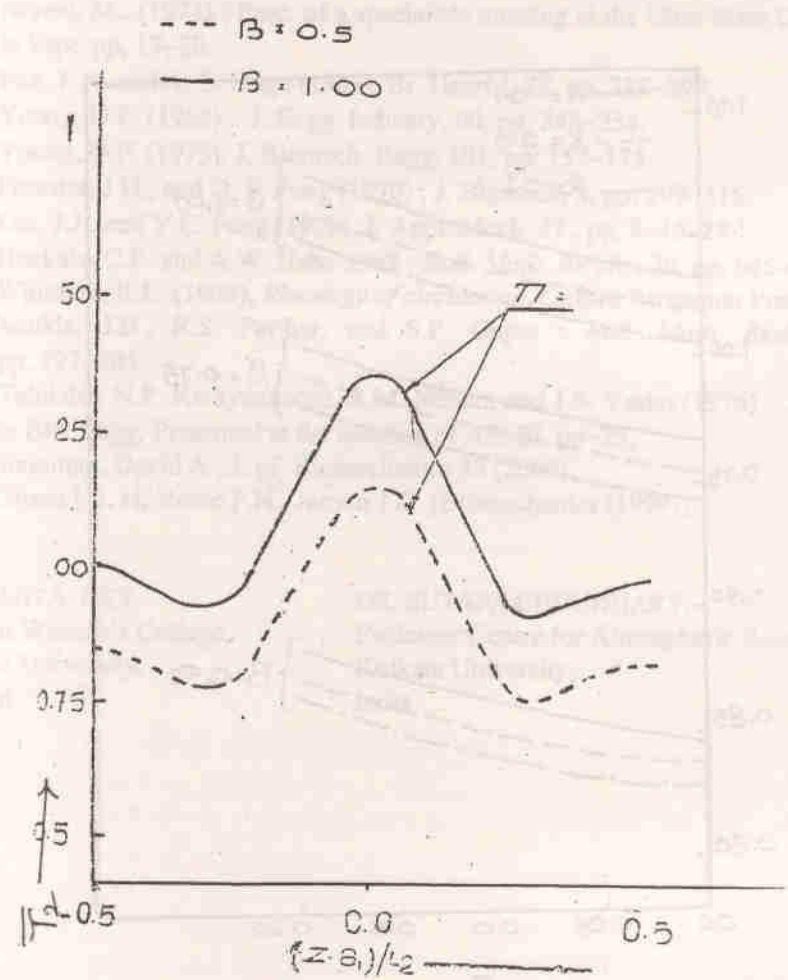

Fig. 1.2. Effect of 'B' and 'n' on $\overline{T}_2$ ($\bar{\delta}_2$ = 0.1; k = 0)

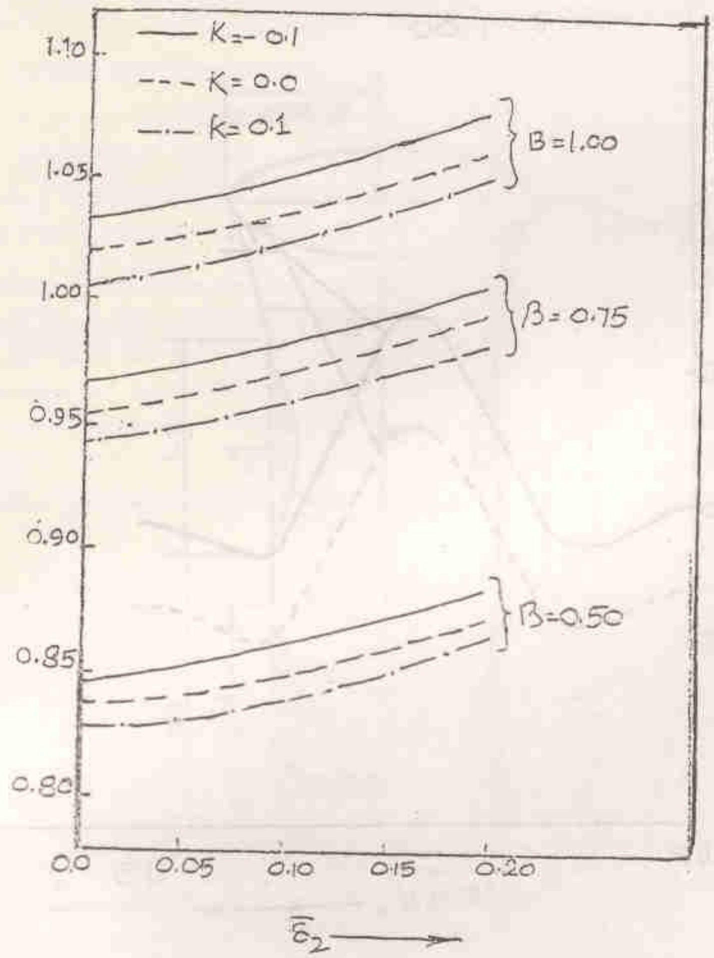

Fig. 1.3 Effect of $\bar{\delta}_2$ on $\bar{\lambda}$ with variation in B and K
(n = 0.15 $\bar{T}_2$(n = 0.15 $\bar{\delta}_1 = 0.1$).

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