The Nepali Math. Sc. Report
Vol. 30, No. 1 & 2, 2010

Common Fixed Point Theorem for Four Mappings in Dislocated Quasi–Metric Spaces

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Abstract: We have proved a common fixed point theorem for four mappings in dislocated metric spaces.

Keywords: dislocated quasi-metric,dq-limit, dq-convergent, dq-cauchy's sequence, fixed point, weakly compatible.

1. Introduction

B.E. Rhoades [2] established various definitions of contractive mappings. In 1992 Banach proved fixed point theorem for contraction mapping in complete metric space. In 1996 Jungck [5] introduced the concept of weakly compatible.

In 2005 F.M. Zeyada, G.H. Hassan, M.A. Ahmed [4] established various definitions of dislocated quasi-metric space. A Isufati [1] proved some fixed point theorems for a single and a pairs of mappings in dislocated metric space. In this paper we proved common fixed point theorem for four maps in dislocated quasi-metric space.

2. Preliminaries

Definition 2.1 [4] Let X be a nonempty set and let $d:X\times X \to [0, \infty) \to [0, \infty)$ be a function satisfying following conditions:

(i) $d(x,y) = d(y,x) = 0$, implies $x = y$,

(ii) $d(x,y) \leq d(x,z)+ d(z,y)$, for all $x,y,z\in X$.

Then d is called a dislocated quasi-metric on X.

If d satisfies d(x,y) = d(y,x), then it is called dislocated metric.

Definition 2.2 [4] A sequence $\{x_n\}$ in dq-metric space (dislocated quasi-metric space) (X,d) is called Cauchy sequences if for given $\varepsilon > 0, \exists$ $n_0 \in N$, such that $\forall$ m,n $\geq n_0$, implies $d(x_m,x_n) < \in$ or $d(x_n,x_m) < \in$ i.e. $\min\{d(x_m, x_n), d(x_n, x_m)\} < \varepsilon$.

Definition 2.3 [4] A sequence $\{x_n\}$ dislocated quasi-converges to x if

$$Lim_{n\to\infty} d(x_n, x) = lim_{n\to\infty} d(x, x_n) = 0$$

In this case x is called a dq-limit of $\{x_n\}$ and we write $x_n \to x$.

Lemma: dq-limit in a dq-metric space are unique.

Definition 2.4 [4] A dq-metric space (X,d) is called complete if every Cauchy sequence in it is a dq-convergent.

Definition 2.5 [4] Let (X,d_1) and (Y,d_2) be dq-metric spaces and let $f{:}X\to Y$ be a function. Then f is continuous to $x_0 \in X$, if for each sequence $\{x_n\}$ which is d_1 -q convergent to x_0, the sequence $\{f(x_n)\}$ is d_2-q convergent to $f(x_0)$ in Y.

Definition 2.6 [4] Let (X,d) be a dq-metric space. A map T: X→X is called contraction if there exists $0 \leq \lambda < 1$ such that

$$d(Tx,Ty) \leq \lambda d(x,y) \text{ for all } x,y \in X.$$

Definition 2.7[9] Let A and S be mapping from a metric space (X,d) into itself. A and S are said to be weakly compatible if they commute at their coincidence points, that is, Ax = Sx for some $x \in X$ implies that ASx = Sax.

Theorem 2.8 [4] Let (X,d) be a dq-metric space and let T: X→X be a continuous contraction mapping. Then T has unique fixed point.

3. Main Results

Theorem 3.1 Let (X,d) be a complete dislocated metric space. Let F,G, S,T : X→X be continuous mapping satisfying:

(i) $S(X) \subseteq G(X), T(X) \subseteq F(X)$ and T(X) or S(X) is a closed subset of X

(ii) The pairs (S,F) and (T,G) are weakly compatible

$$d(Sx,Ty) \le h \max\left\{ d(Fx,Gy), d(Fx,Sx), d(Gy,Ty), \frac{d(Fx,Ty)+d(Gy,Sx)}{2} \right\}$$

for all x,y$\in X$ and 0<h<1. Then f , g, s and t have common fixed point.

Proof: Suppose x_0 is an arbitrary point of X and define the sequence $\{y_n\}$ in X such that

$$Y_{2n} = Sx_{2n} = Gx_{2n+1}$$

$$Y_{2n+1} = Tx_{2n+1} = Fx_{2n+2}$$

Consider

$$d(y_{2n}, y_{2n+1}) = d(Sx_{2n}, Tx_{2n+1})$$

$$\le h \max\left\{ d(Fx_{2n}, Gx_{2n+1}), d(Fx_{2n}, Sx_{2n}), d(Gx_{2n+1}, Tx_{2n+1}), \frac{d(Fx_{2n}, Tx_{2n+1}) + d(Gx_{2n+1}, Sx_{2n})}{2} \right\}$$

$$= h \max\left\{ d(y_{2n-1}, y_{2n}), d(y_{2n-1}, y_{2n}), d(y_{2n}, y_{2n+1}), \frac{d(y_{2n-1}, y_{2n}) + d(y_{2n}, y_{2n})}{2} \right\}$$

$$d(y_{2n}, y_{2n+1}) \le h d(y_{2n-1}, y_{2n})$$

Similarly

$$d(y_{2n-1}, y_{2n}) \le h d(y_{2n-2}, y_{2n-1})$$

and so $d(y_{2n}, y_{2n+1}) \le h^2 d(y_{2n-2}, y_{2n-1})$

In this way we have

$$d(y_{2n}, y_{2n+1}) \le h^n d(y_1, y_0)$$

Since o<h<1, as $h^n \to 0$ as $n \to \infty$. Thus $\{y_n\}$ is Cauchy sequence in a complete dislocated metric space X. There exists a point u$\in X$ such that $\{y_n\} \to$u.

Therefore the subsequences $\{Sx_{2n}\} \to u$, $\{Tx_{2n+1}\} \to u$, $\{Gx_{2n+1}\} \to u$ and $\{Fx_{2n+2}\} \to u$.

Assume that G(X) is a closed subset of X. Then $\exists$ v$\in X$ such that Gv = u.

If Tv $\neq$u then, we obtain

$$d(Sx_{2n}, Tv)$$
$$\leq h \max\left\{ d(Fx_{2n}, Gv), d(Fx_{2n}, Sx_{2n}), d(Gv, Tv), \frac{d(Fx_{2n}, Tv) + d(Gv, Sx_{2n})}{2} \right\}$$
$$= h \max\left\{ d(y_{2n-1}, Gv), d(y_{2n-1}, y_{2n}), d(Gv, Tv), \frac{d(y_{2n-1}, Tv) + d(Gv, y_{2n})}{2} \right\}$$

As $n \to \infty$, we get

$$d(u, Tv) \leq h \max\left\{ d(u, Gv), d(u,u), d(Gv, Tv), \frac{d(u, Tv) + d(Gv, u)}{2} \right\}$$
$$< d(u,Tv)$$

It follows that Tv = u = Gv. Since B and T are weakly compatible,

we have TGv = GTv and so Tu = Gu.

If u ≠ Bu we get

$$d(Sx_{2n}, Tu)$$
$$\leq h \max\left\{ d(Fx_{2n}, Gu), d(Fx_{2n}, Sx_{2n}), d(Gu, Tu), \frac{d(Fx_{2n}, Tu) + d(Gu, Sx_{2n})}{2} \right\}$$
$$= h \max\left\{ d(y_{2n-1}, Gu), d(y_{2n-1}, y_{2n}), d(Gu, Tu), \frac{d(y_{2n-1}, Tu) + d(Gu, y_{2n})}{2} \right\}$$

As $n \to \infty$, we get

$$d(u, Tu) \leq h \max\left\{ d(u, Gu), d(u,u), d(Gv, Tu), \frac{d(u, Tu) + d(Gu, u)}{2} \right\}$$
$$< d(u,Tu)$$

and so Tu = u

since $T(X) \subseteq F(X)$, $\exists$ w$\in X$ such that Fw = u.

If Sw ≠ u, we have

$$d(Sw, Tu) \leq h \max\left\{ d(Fw, Gu), d(Fw, Sw), d(Gu, Tu), \frac{d(Fw, Tu) + d(Gu, Sw)}{2} \right\}$$

And it follows that

$$d(Sw, u) \leq h \max\left\{ d(Fw, u), d(Fw, Sw), d(u, Tu), \frac{d(Fw, Tu) + d(u, Sw)}{2} \right\}$$
$$< d(Sw,u)$$

This implies that Sw = u and hence Sw = Fw = u.

Since S and F are weakly compatible

SFw = FSw and so

Su = Fu.

If Su ≠ u then, we get

d(Su,u) = d(Su,Tu)

$$\leq h \max\left\{ d(Fu,u), d(Fu,Su), d(u,Tu), \frac{d(Fu,u)+d(u,Su)}{2} \right\}$$

$$= d(Su,u)$$

And so Su = u. Thus Su = Tu = Fu = Gu = u, that is u is a common fixed point for S,T,F,G.

Uniqueness of common fixed point:

Let u and v be a common fixed point of F , G, S and T. Then

$$d(u,v) \leq d(Su,Tv) \leq h \max\left\{ d(Fu,Gv), d(Fu,Su), d(Gv,Tv), \frac{d(Fu,Tv)+d(Gv,Su)}{2} \right\}$$

$$\leq h \max\left\{ d(u,v), d(u,u), d(v,v), \frac{d(u,v)+d(v,u)}{2} \right\}$$

$$\leq h \max\{ d(u,v), d(u,u), d(v,v), \}$$

Replacing v by u, we get d(u,v) ≤ h d(u,u). Since 0<h<1, we have d(u,u) = 0. Similarly we have d(v,v) = 0.

In this way we have d(u,v) ≤ h d(u,v). Since 0<h<1,

we have d(u,v) = 0.

Similarly, we have d(v,u) = 0 and so u = v.

Hence u is unique common fixed point of F, G, S and T.

Hence the proof is completed.

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