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Interdependent Machining System with Spares, Controllable Arrival Rates and Additional Repairmen

M. JAIN, G.C. SHARMA AND K.P.S. BAGHEL

Abstract: The provision of spares and repair facility has utmost significance for machining systems of manufacturing / production processes. This paper deals with machine repair system consisting of operating units along with spare units under the care of a repair facility having permanent as well as removable additional repairmen. It is considered that machines are interdependent. The breakdown times and renovation times of units are exponentially distributed. The repair facility controls the breakdown units by rendering the service in FCFS order. The mean queue length is obtained by using recursive method.

Key words: Machine repair, FCFS, Interdependent, Spares, Repairmen, Mean queue length.

1. Introduction

The applicability of machines can be realised in any manufacturing production organization to fulfil the desired requirements of the jobs. To design a better machining system, the system designer must have good technical knowledge of performance prediction apart from sufficient knowledge of various designs issues.

Several queue theorist have studied machining systems in different frameworks. Gaglio and Wagner (1964) gave an approximate solution for three-machine scheduling problem. Wang (1990) studied machine repair problem with single service station subject to breakdowns and analysed the profit of the system. The cost analysis of the machine repair problem with non-reliable service stations was investigated by Wang and Hsu (1995). Two-machine production line with state-dependent rate was considered by Jain (1999). Jain et al. (2002) also studied a flexible manufacturing system to evaluate its performance indices. A little bit literature whatsoever is available in which the skilful research workers conducted their research works incorporating the provision of spares provisioning, is worth mentioning. Some useful works in this direction are also made by many researchers.

Natarajan (1968) analysed a reliability problem with spares and multiple repair facilities. Srinivasan and Gopalan (1973) studied a two-unit system having warm standby spares and estimated availability and reliability of the system. Jain (1997) analysed state-dependent (m.M) machine repair problem with spares.

The facility of additional repairmen may be cost effective to improve the system availability in particular when there is sufficient backlog of failed units in the system M/M/C/K/N machine repair problem with balking, reneging, spares and additional repairman was studied by Jain et al. (2000a). Jain et al. (2000b) also considered machine repair problem with spares, reneging, additional repairman and two modes of failure. Jain and Baghel (2001) considered a multi-component repairmen repairable system with facilitating spare parts and state-dependent rates.

Agnihothri (1989) studied a machining problem with ingenuous repairmen and interrelationship between their performance measures. Gupta (1995) developed a queueing system with interrelationship between controlled arrival and service rates. An M/M/l interdependent queueing system having controllable arrival rates was analysed by Rao et al. (2000). Recently, M/M/c interdependent queueing model with controllable arrival rates was studied by Begum and Maheswari (2002).

In present investigation, we consider a multi-component system having correlated failure and repair rates. We have incorporated the provision of spares and two additional repairmen in our model. The whole paper is ramified into four sections. In section 2, we describe assumptions and notations to describe the mathematical model. The governing equations and their solutions in explicit form are given in section 3. The average queue length is obtained. The last section 4 includes the scope of future research work and concluding remarks.

2. System Postulates with Notations

We consider an interdependent machining system consisting of M operating machines, R permanent repairmen and S Warm spares. There is provision of two additional repairmen to increase efficiency of the system in case of heavy traffic of failed machines. The following assumptions are made to elaborate the model as :

- There is a correlation between breakdown times and repair times of machines, which are exponentially distributed with mean rate $1/\lambda$ and $1/\mu$ respectively.
- Whenever a machine fails, it is sent to renovate in repair facility in order of their breakdowns.
- After renovation, the machine is as good as at the time of failure and joins the group of standby or functioning machines.
- The spare units are assumed to fail according to exponential distribution with rate α
- When all the spares are used, the operating machines fail with degraded failure rate λ.

- To reduce the queue size, the first additional repairman is provided at threshold level T_0+1 and preceded till queue length drops to Q_0.
- The second additional repairman starts repair at threshold level T_1+1 and continue till queue length drops to Q_1

3. The Equations and Analysis

Denote

$$\Lambda_n = M(\lambda-e)+(S-n)(\alpha-e) \qquad n<S$$
$$\Lambda_n = (M+S-n)+(\lambda'-e) \qquad n\geq S$$

and

$$R_j = R(\mu-e)+(\mu_j-e)$$

The steady-state equations governing the model are given as follows :

(1) $\Lambda_0 P_0(0) = (\lambda-e)P_1(0)$

(2) $[\Lambda_n + n(\mu-e)]P_n(0) = \Lambda_{n-1}P_{n-1}(0)+(n+1)(\mu-e)P_{n+1}(0), \qquad 1\leq n<R$

(3) $[\Lambda_n + R(\mu-e)]\,P_n(0) = \Lambda_{n-1}P_{n-1}(0)+R(\mu-e)P_{n+1}(0), \qquad 1\leq n<R$

(4) $[\Lambda'_S + R(\mu-e)]\,P_S(0) = \Lambda_{S-1}P_{S-1}(0)+R(\mu-e)P_{S+1}(0)$

(5) $[\Lambda'_n + R(\mu-e)]P_n(0) = \Lambda'_{n-1}P_{n-1}(0)+R(\mu-e)P_{n+1}(0), \qquad S<n<Q_0$

(6) $[\Lambda'_{Q_0} + R(\mu-e)]P_{Q_0}(0) = \Lambda'_{Q_0-1}P_{Q_0-1}(0)+R(\mu-e)P_{Q_0+1}(0)+R'_1 P_{Q_0+1}(1)$

(7) $[\Lambda'_n + R(\mu-e)]P_n(0) = \Lambda'_{n-1}P_{n-1}(0)+R(\mu-e)P_{n+1}(0), \qquad Q_0+1\leq n<T_0$

(8) $[\Lambda'_{T_0} + R(\mu-e)]P_{T_0}(0) = \Lambda'_{T_0-1}P_{T_0-1}(0)$

(9) $[\Lambda'_{Q_0+1}+R'_1]P_{Q_0+1}(1) = R'_1 P_{Q_0+2}(1)$

(10) $[\Lambda'_n+R'_n]P_n(1) = \Lambda'_{n-1}P_{n-1}(1)+R'_1P_{n+1}(1), \qquad Q_0+2<n<T_0+1$

(11) $[\Lambda'_{T_0+1}+R'_1]P_{T_0+1}(1) = \Lambda'_{T_0}P_{T_0}(0)+\Lambda'_{T_0}P_{T_0}(1)+R'_1P_{T_0+2}(1)$

(12) $[\Lambda'_n+R'_1]P_n(1) = \Lambda'_{n-1}P_{n-1}(1)+R'_1P_{n+1}(1), \qquad T_0+2\leq n<Q_1$

(13) $[\Lambda'_{Q_1}+R'_1]P_{Q_1}(1) = \Lambda'_{Q_1-1}P_{Q_1-1}(1)+R'_1P_{Q_1+1}(1)+R'_2P_{Q_1+1}(2)$

(14) $[\Lambda'_n+R'_1]P_n(1) = \Lambda'_{n-1}P_{n-1}(1)+R'_1P_{n+1}(1), \qquad Q_1+1<n<T_1$

(15) $[\Lambda'_{T_1}+R'_1]P_{T_1}(1) = \Lambda'_{T_1-1}P_{T_1-1}(1)$

(16) $[\Lambda'_{Q_1+1}+R'_2]P_{Q_1+1}(2) = R'_2 P_{Q_1+2}(2)$

(17) $[\Lambda'_n+R'_2]P_n(2) = \Lambda'_{n-1}P_{n-1}(2)+R'_2P_{n+1}(2), \qquad Q_1+2\leq n<T_1+1$

(18) $[\Lambda'_{T_1+1}+R'_2]P_{T_1+1}(2) = \Lambda'_{T_1}P_{T_1}(1)+\Lambda'_{T_1}P_{T_1}(2)+R'_2P_{T_1+2}(2)$

(19) $[\Lambda'_n+R'_2]P_n(2) = \Lambda'_{n-1}P_{n-1}(2)+R'_2P_{n+1}(2), \qquad T_1+2\leq n\leq M+S$

(20) $\Lambda'_{M+S-1}\, P_{M+S-1}(2) = R'_2\, P_{M+S}(2)$

On solving equations (1)-(4), we have

$$(21)\qquad P_n(0) = \begin{cases} \dfrac{\prod_{i=0}^{n-1}\Lambda_i}{n!(\mu-e)^n} P_0(0), & 1 \le n \le R \\[2ex] \dfrac{\prod_{i=0}^{n-1}\Lambda_i}{R!\,R^{n-R}(\mu-e)^n} P_0(0), & R < n \le S \\[2ex] \dfrac{\prod_{i=0}^{S-1}A_i \prod_{i=0}^{n-1}\Lambda'_i}{R!\,R^{n-R}(\mu-e)^n} P_n(0), & S < n \le Q_0 \end{cases}$$

Substituting the values of $P_{Q_0-1}(0)$ and $P_{Q_0}(0)$ from equation (21) in equations (5)-(7), we get

$$(22)\qquad P_{Q_0+1}(0) = \frac{\prod_{i=0}^{S-1}\Lambda_i \prod_{i=S}^{Q_0}\Lambda'_i}{R!\,R^{Q_0+1-R}(\mu-e)^{Q_0+1}} P_{Q_0}(0) - \left[1 + \frac{(\mu'_1 - e)}{R(\mu-e)}\right] P_{Q_0+1}(1)$$

and in general we have

$$(23)\qquad P_n(0) = \frac{\prod_{i=0}^{S-1}\Lambda_i \prod_{i=S}^{n-1}\Lambda'_i}{R!\,R^{n-R}(\mu-e)^n} P_n(0) - \left[1 + \sum_{i=Q_0+1}^{n-1}\prod_{j=i}^{n-1}\frac{\Lambda'_j}{R(\mu-e)}\right] \times$$

$$\times\left[1 + \frac{(\mu_1 - e)}{R(\mu-e)}\right] P_{Q_0+1} \;\cdots\; (1)$$

$$Q_0 < n \le T_0$$

Now substituting the value of $P_{T_0-1}(0)$ and $P_{T_0}(0)$ from equation (23), equation (8) yields

(24) $P_n(0) =$

$$\frac{\prod_{i=0}^{S-1}\Lambda_i \prod_{i=S}^{T_0}\Lambda_i}{R!\,R^{T_0-R}(\mu-e)^{T_0}\left[\sum_{i=Q_0+1}^{T_0-1}\prod_{j=1}^{T_0-1}\frac{\Lambda'_i}{R(\mu-e)}\sum_{i=Q_0+1}^{T_0-2}\prod_{j=1}^{T_0-2}\frac{\Lambda'_i}{R(\mu-e)}\right]\left[1 + \frac{(\mu_1 - e)}{R(\mu-e)}\right]} P_0(0)$$

$$= WP_0(0)$$

where

$$W = \frac{\prod_{i=0}^{S-1}\Lambda_i \prod_{i=S}^{T_0}\Lambda'_i}{R!\,R^{T_0-R}(\mu-e)^{T_0}\left[\sum_{i=Q_0+1}^{T_0-1}\prod_{j-1}^{T_0-1}\frac{\Lambda'_i}{R(\mu-e)} - \sum_{i=Q_0+1}^{T_0-2}\prod_{j-1}^{T_0-2}\frac{\Lambda'_i}{R(\mu-e)}\right]\left[1+\frac{(\mu_1-e)}{R(\mu-e)}\right]}$$

Now equation (23) reduces to

$$\text{(25)}\quad P_n(0)=\left[\frac{\prod_{i=0}^{S-1}\Lambda_i\prod_{i=S}^{n-1}\Lambda'_i}{R!\,R^{n-R}(\mu-e)^n}-W\left\{\sum_{i=Q_0+1}^{n-1}\prod_{j=i}^{n-1}\frac{\Lambda'_j}{R(\mu-e)}\right\}\left[1+\frac{(\mu_1-e)}{R(\mu-e)}\right]\right]P_n(0)$$

$$Q_0 < n < T_0$$

Using equations (9), (10) and (34), we find

$$\text{(26)}\quad P_n(0) = \left[1+\sum_{i=Q_0+1}^{n-1}\prod_{j=i}^{n-1}\frac{\Lambda'_j}{R'_1}\right]W\,P_0(0) \qquad Q_0+2 \le n < T_0+1$$

Equation (11) provides

$$P_{T_0+2}(1)=\left[\left(1+\frac{\Lambda'_{T_0+1}}{R'_1}\right)\left\{1+\sum_{i=Q_0+1}^{T_0}\prod_{j=i}^{T_0}\frac{\Lambda'_j}{R'_1}\right\}-\frac{\Lambda'_{T_0}}{R'_1}\left\{1+\sum_{i=Q_0+1}^{T_0-1}\prod_{j=i}^{T_0-1}\frac{\Lambda'_j}{R'_1}\right\}\right]W\,P_0(0)$$

$$\text{(27)}\quad -\frac{\Lambda'_{T_0}}{R'_1}\left[\frac{\prod_{i=0}^{S-1}\Lambda_i\prod_{i=S}^{T_0-1}\Lambda'_i}{R!\,R^{T_0-R}(\mu-e)^n}-W.\prod_{i=Q_0+1}^{T_0-1}\left[1+\frac{\Lambda'_1}{R(\mu-e)}\right]\left[1+\frac{(\mu_1-e)}{R(\mu-e)}\right]\right]P_n(0)$$

With the help of equation (12), we have

$$\text{(28)}\quad P_{T_0+3}(1)=\left(1+\frac{\Lambda'_{T_0+2}}{R'_1}\right)P_{T_0+2}(1)-\frac{\Lambda'_{T_0+1}}{R'_1}P_{T_0+1}(1)$$

...

in general equation (12) gives

$$\text{(29)}\quad P_n(1)\ \left(1+\sum_{i=T_0+2}^{n-1}\prod_{j=i}^{n-1}\frac{\Lambda'_j}{R'_1}\right)P_{T_0+2}(1)-\frac{\Lambda'_{T_0+1}}{R'_1}\left(1+\sum_{i=T_0+3}^{n-1}\prod_{j=i}^{n-1}\frac{\Lambda'_j}{R'_1}\right)P_{T_0+1}(1)\ ,$$

$$T_0+4 \le n < Q_1$$

Equation (13) provides

$$(30) \qquad P_{T_1+1}(1) = \left(1+\frac{\Lambda'_{Q_1}}{R'_1}\right) P_{Q_1}(1) - \frac{\Lambda_{Q_1-1}}{R'_1} P_{Q_1-1}(1) \frac{R'_2}{R'_1} P_{Q_1+1}(2)$$

Also equation (13) yields to

$$(31) \qquad P_{Q_1+2}(1) = \left(1+\frac{\Lambda'_{Q_1+1}}{R'_1}\right) P_{Q_1+1}(1) - \frac{\Lambda'_{Q_1}}{R'_1} P_{Q_1}(1)$$

Similarly in general, we find

$$(32) \qquad P_n(1) = \left(1+\sum_{i=Q_1+1}^{n-1}\prod_{j=i}^{n-1}\frac{\Lambda'_j}{R'_1}\right) P_{Q_1+1}(1) - \frac{\Lambda'_{Q_1}}{R'_1}\left(1+\sum_{i=Q_1+2}^{n-1}\prod_{j=i}^{n-1}\frac{\Lambda'_j}{R'_1}\right) P_{Q_1}(1)$$

$$Q_1+3 \le n < T_1 - 1$$

On solving equation (15), we have

$$(33) \qquad P_{T_1}(1) = \frac{\Lambda_{T_1-1}}{\Lambda'_{T_1} R'_1} P_{T_1-1}(1)$$

From equation (16), we find

$$(34) \qquad P_{Q_1+2}(2) = \left(1+\frac{\Lambda'_{Q_1+1}}{R'_2}\right) P_{Q_1+1}(2)$$

Now equation (17) gives

$$(35) \qquad P_{Q_1+3}(2) = \left(1+\frac{\Lambda'_{Q_1+2}}{R'_2}\right) P_{Q_1+2}(2) - \frac{\Lambda'_{Q_1+1}}{R'_2} P_{Q_1+1}(2)$$

and in general, we have

$$(36) \qquad P_n(2) = \left(1+\sum_{i=Q_1+2}^{n-1}\prod_{j=i}^{n-1}\frac{\Lambda'_j}{R'_2}\right) P_{Q_1+2}(2) - \frac{\Lambda'_{Q_1+1}}{R'_2}\left(1+\sum_{i=Q_1+3}^{n-1}\prod_{j=i}^{n-1}\frac{\Lambda'_j}{R'_2}\right) P_{Q_1+1}(2)$$

$$Q_1+4 \le n < T_1 + 1$$

The equation (18) yields

$$(37) \qquad P_{T_1+2}(2) = \left(1+\frac{\Lambda_{T_1+1}}{R'_2}\right) P_{T_1+1}(1) - \frac{\Lambda_{T_1}}{R'_2} P_{T_1}(2) - \frac{\Lambda_{T_1}}{R'_2} P_{T_1}(1)$$

Further equation (19) provides

$$(38) \qquad P_{T_1+3}(2) = \left(1+\frac{\Lambda'_{T_1+2}}{R'_2}\right) P_{T_1+2}(1) - \frac{\Lambda'_{T_1+1}}{R'_2} P_{T_1+1}(2)$$

and in general we find

(39) $$P_n(2) = \left(1 + \sum_{i=T_1+2}^{n-1} \prod_{j=i}^{n-1} \frac{\Lambda'_j}{R'_2}\right) P_{T_1+2}(2) - \frac{\Lambda'_{T_1+1}}{R'_2}\left(1 + \sum_{i=T_1+3}^{n-1} \prod_{j=i}^{n-1} \frac{\Lambda'_j}{R'_2}\right) P_{T_1+1}(2)$$

$$T_1 + 4 \le n < M + S - 1$$

Now equation (20) and (39) give

$$P_{M+S}(2) - \frac{\Lambda'_{T_1+1}}{R'_2}\left(1 + \sum_{i=T_1+2}^{M+S-2} \prod_{j=i}^{M+S-2} \frac{\Lambda'_j}{R'_2}\right) P_{T_1+2}(2)$$

(40) $$\frac{\Lambda_{T_1+1}}{R'_2}\left(1 + \sum_{i=T_1+3}^{M+S-2} \prod_{j=i}^{M+S-2} \frac{\Lambda'_j}{R'_2}\right) P_{T_1+2}(2)$$

Mean Queue Length

The mean queue length is obtained as

(41) $$L = \sum_{n=0}^{T_0} nP_n(0) + \sum_{n=Q_0+1}^{T_1} nP_n(1) + \sum_{n=Q_1+1}^{M+S} nP_n(2)$$

4. Discussion

In this paper, steady state queue size distribution for repairable machining system with spare provisioning has been determined using recursive method. By using spare parts support and controllable arrival rates in the presence of permanent as well as additional repairmen, we have studied more versatile problem of machining system. For higher productivity and efficiency, the conventional machining system has the provision of spare part support. The additional repairmen may be helpful to reduce the backlog of the system which is essential for economical as well as reliability requirement viewpoints.

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M. JAIN, G. C. SHARMA AND K. P. S. BAGHEL
Department of Mathematics, Institute of Basic Science,
Khandari, Agra – 282002