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Mathematical Model of Slider Bearing

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Abstract: To reduce tear and wear of machinery lubrication is essential. Lubricants form a layer between two surfaces preventing direct contact and reduce friction between moving parts and hence reduce wear. The choice of lubricant is important for a given application. In this model the lubrication of the slider bearing is studied. A simple slider bearing has two plates of given profile separated by a gap between the plates is filled with the lubricant. One of the plates is fixed and other is moving horizontally. Due to the viscosity of the lubricant, motion of the plate's results in work done on the lubricant increasing the temperature. This study will be helpful in finding the condition under which the safe operation of the bearing is ensured. That is, in finding the condition under which the temperature of the lubricant is lower than the ignition temperature. When the variable viscosity is considered the case becomes complicated. Further investigations are necessary.

Key words: Navier-Stokes Equations, dimensional analysis, scaling, load.

1. Slider Bearing with Parallel Plates

The incompressible Navier- Stokes equation is

$$\frac{\partial \mathrm{u}}{\partial t} + \mathrm{u}\nabla \mathrm{u} - f - \upsilon\Delta \mathrm{u} + \frac{\nabla p}{\rho} = 0 \qquad (1)$$

$$\nabla . \mathrm{u} = 0 \qquad (2)$$

where (1) represents the conservation of momentum. $\mathrm{u} = (u, w)$ be a vector, p is pressure and ρ is the density, υ is kinematic viscosity. The body force f is usually absent and other parameters remain constant.

Let $u(x, z, t)$ and $w(x, z, t)$ be the components of velocity of the fluids in horizontal and vertical directions, respectively then by (1) and (2)

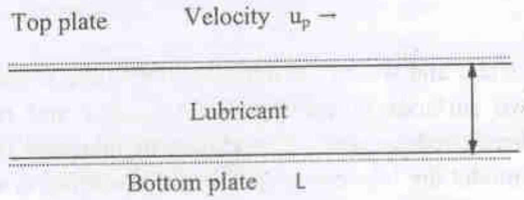
$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} + \frac{1}{\rho}\frac{\partial p}{\partial x} = \upsilon\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right) \qquad (3)$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z} + \frac{1}{\rho}\frac{\partial p}{\partial z} = \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 w}{\partial z^2}\right) \tag{4}$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \tag{5}$$

By imposing the boundary conditions

$$(u, w) = (0, 0)\ at\ z = 0;$$
$$(u, w) = (u_p, 0)\ at\ z = l$$



Bearing with parallel plates

Here, u_p is the typical horizontal velocity of the plate and l is the typical separation between the plates which is shown as in figure. Let the following parameters for bearing used are :

$L = 5cm$, $l = 5\mu m$, $u_p = 1m/sec$, $\rho = 1 \times 10^3 Kgm^{-3}$, $\mu = 1 \times 10^{-4} m^2/sec$

To non- dimensionalize the equations parameters are scaled as

$$x = \bar{x}L\ , z = \bar{z}l\ , u = \bar{u}u_p\ , w = \bar{w} \in u_p\ , \ t = \bar{t}\, L/u_p\ , p = \bar{p}P$$

From (3) and (4), we get

$$\frac{u_p^2}{L}\frac{\partial \bar{u}}{\partial \bar{t}} + \frac{\bar{u}u_p^2}{L}\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\bar{w}\epsilon u_p^2}{l}\frac{\partial \bar{u}}{\partial \bar{z}} + \frac{1}{\rho}\frac{P}{L}\frac{\partial \bar{p}}{\partial \bar{x}} = \nu\left(\frac{u_p}{L^2}\frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \frac{u_p}{l^2}\frac{\partial^2 \bar{u}}{\partial \bar{z}^2}\right) \tag{6}$$

$$\frac{\epsilon u_p^2}{L}.\frac{\partial \bar{w}}{\partial \bar{t}} + \frac{\bar{u}\epsilon u_p^2}{L}.\frac{\partial \bar{w}}{\partial \bar{x}} + \bar{w}\frac{\epsilon^2 u_p^2}{l}\frac{\partial \bar{w}}{\partial \bar{z}} + \frac{1}{\rho}\frac{P}{L}\frac{\partial \bar{p}}{\partial \bar{z}} = \nu\left\{\frac{\epsilon u_p}{L^2}\frac{\partial^2 \bar{w}}{\partial \bar{x}^2} + \frac{\epsilon u_p}{l^2}\frac{\partial^2 \bar{w}}{\partial \bar{z}^2}\right\} \tag{7}$$

Multiplying (6) by $\frac{L}{u_p^2}$ and (7) by $\frac{L}{\epsilon u_p^2}$ and setting, $\epsilon = \frac{l}{L}$ is typically 1×10^{-4} . P is undecided scaling factor for the pressure. Eliminating the terms which have small coefficients as compared to $\frac{1}{\epsilon}$, from (6)

$$\frac{P}{\rho u_p^2}\frac{\partial \bar{p}}{\partial \bar{x}} = \frac{\nu L}{l^2 u_p}\frac{\partial^2 \bar{u}}{\partial \bar{z}^2}$$

Here, P is chosen such that

$$\frac{P}{\rho u_p^2} = \frac{\nu L}{l^2 u_p}$$

$$P = \frac{\rho v u_p L}{l^2}$$

$$P = \frac{\mu u_p L}{\cdot l^2} \tag{8}$$

where $\mu = \rho v$

With the choice of P, we get from (6)

$$\frac{\partial \bar{p}}{\partial \bar{x}} = \frac{\partial^2 \bar{u}}{\partial \bar{z}^2} \tag{9}$$

and from (7)

$$\frac{1}{\rho} \cdot \frac{PL}{\epsilon u_p^2} \frac{\partial \bar{p}}{\partial \bar{z}} = 0$$

$$\therefore \frac{\partial \bar{p}}{\partial \bar{z}} = 0 \tag{10}$$

From (5), we obtain

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{w}}{\partial \bar{z}} = 0 \tag{11}$$

The boundary condition for $\bar{u}$ and for $\bar{w}$

$\bar{u} = 0$ at $\bar{z} = 0$ and $\bar{u} = 1$ at $\bar{z} = 1$

$\bar{w} = 0$ at $\bar{z} = 0$ and $\bar{w} = 0$ at $\bar{z} = 1$,

Since the pressure $\bar{p}$ is independent of $\bar{z}$ then from equation (10)

$$\frac{\partial \bar{p}}{\partial \bar{z}} = 0$$

Integrating, $\bar{p} = \emptyset(\bar{x})$ with the boundary conditions;

$\bar{p} = 0$ at $\bar{x} = 0$

$\bar{p} = 0$ at $\bar{x} = 1$

Integrating first we obtain,

$$\frac{\partial \bar{u}}{\partial \bar{z}} = \frac{\partial \emptyset(\bar{x})}{\partial \bar{x}} \bar{z} + c_1$$

Again integrating,

$$\bar{u} = \frac{\partial \emptyset(\bar{x})}{\partial \bar{x}} \cdot \frac{\bar{z}^2}{2} + c_1 \bar{z} + c_2 \tag{12}$$

where c_1 and c_2 are constants.

With the boundary condition $\bar{u} = 0$ at $\bar{z} = 0$ and $\bar{u} = 1$ at $\bar{z} = 1$

We get, $c_2 = 0$ and $c_1 = 1 - \frac{1}{2}\frac{d\emptyset}{d\bar{x}}$

$$\therefore \quad \bar{u} = \left(\frac{\bar{z}^2 - \bar{z}}{2}\right)\frac{d\emptyset}{d\bar{x}} + \bar{z}$$

Again from equation (11), $\frac{\partial \bar{w}}{\partial \bar{z}} = -\frac{\partial \bar{u}}{\partial \bar{x}}$

$$\frac{\partial \bar{w}}{\partial \bar{z}} = -\frac{\partial}{\partial \bar{x}}\left[\left(\frac{\bar{z}^2-\bar{z}}{2}\right)\frac{d\emptyset}{d\bar{x}} + \bar{z}\right]$$

$$\frac{\partial \bar{w}}{\partial \bar{z}} = -\frac{\partial}{\partial \bar{x}}\left[\left(\frac{\bar{z}^2-\bar{z}}{2}\right)\frac{d\emptyset}{d\bar{x}} + \bar{z}\right]$$

$$\therefore \bar{w} = -\left[\frac{\bar{z}^3}{6} - \frac{\bar{z}^2}{4}\right]\frac{d^2\emptyset}{d\bar{x}^2} + c_3$$

Where c_3 is constant of integration with the boundary conditions;
$\bar{w} = 0$ at $\bar{z} = 0$ and $\bar{z} = 1$, we have

$$c_3 = 0 \text{ and } c_3 = -\frac{1}{12}\frac{d^2\emptyset}{d\bar{x}^2}$$

$$\therefore \quad \frac{d^2\emptyset}{d\bar{x}^2} = 0$$

Integrating we get,

$$\frac{d\emptyset}{d\bar{x}} = c_4$$

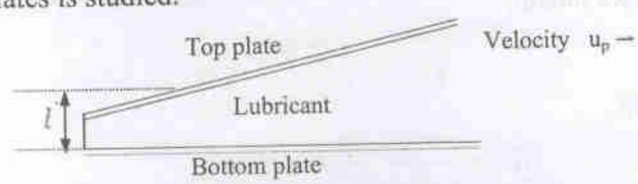
$$\therefore \quad \emptyset = c_4\,\bar{x} + c_5$$

With the boundary conditions on $\emptyset$ begin zero, we must have that $\bar{p} = 0$.

Here, it is shown that the pressure is zero, the bearing with parallel plates can support no load. Therefore, it is not helpful in physical problem. But this problem acts as a clue for the further work.

2. Slider Bearing with Non-parallel Plates

When a bearing with parallel plates cannot support the load, Slider bearing with non-parallel plates is studied.



Bearing with non-parallel plates

We assume that the bottom plate is flat and the top plate is given by $z = h(x)$. Other parameters remain the same as bearing with parallel plates.
We scale the above equations (3), (4) and (5) with

$$x = \bar{x}L\,,\ z = \bar{z}l\,,\ u = u\bar{u}_p,\ w = \bar{w} \in u_p, h(x) = l\bar{h}\,(\bar{x})$$

$$t = \bar{t}\frac{l}{u_p}, p = \bar{p}P$$

We obtain the non-dimensional equations, (9), (10) and (11)

Imposing the boundary conditions

$$\bar{u} = 0 \; at \; \bar{z} = 0$$
$$\bar{u} = 1 \; at \; \bar{z} = \bar{h}(\bar{x})$$

From (12) we get,

$$0 = \frac{d\emptyset}{d\bar{x}}.0 + c_1 0 + c_2 \quad \therefore \; c_2 = 0$$

and $$1 = \frac{d\emptyset}{d\bar{x}}\frac{\bar{h}^2(\bar{x})}{2} + c_1\bar{h}(\bar{x})$$

$$\therefore \quad c_1 = \frac{1}{\bar{h}(\bar{x})} - \frac{1}{2}\frac{d\emptyset}{d\bar{x}}\bar{h}(\bar{x})$$

Thus.

$$\bar{u} = \frac{d\emptyset}{d\bar{x}}\left[\frac{\bar{z}^2}{2} - \frac{\bar{z}\bar{h}}{2}\right] + \frac{\bar{z}}{\bar{h}} \tag{13}$$

Denoting $\frac{d\emptyset}{d\bar{x}}$ by $\emptyset'(\bar{x})$. Now from equation

$$\frac{\partial \bar{w}}{\partial \bar{z}} = -\frac{\partial \bar{u}}{\partial \bar{x}} = -\frac{\partial}{\partial \bar{x}}\left[\left(\frac{\bar{z}^2 - \bar{z}\bar{h}}{2}\right)\frac{d\emptyset}{d\bar{x}} + \frac{\bar{z}}{\bar{h}}\right]$$

$$= -\left[\frac{\bar{z}^2 - \bar{z}\bar{h}}{2}\emptyset''(\bar{x}) - \frac{\bar{z}}{2}\frac{d\bar{h}}{d\bar{x}}\emptyset'(x) + \frac{\bar{h}.0 - \bar{z}\frac{d\bar{h}}{d\bar{x}}}{\bar{h}^2}\right]$$

$$\frac{\partial \bar{w}}{\partial \bar{z}} = -\left[\frac{\bar{z}^2 - \bar{z}\bar{h}}{2}\emptyset''(\bar{x}) - \frac{\bar{z}}{2}\frac{d\bar{h}}{d\bar{x}}\emptyset'(x) - \frac{\bar{z}\frac{d\bar{h}}{d\bar{x}}}{\bar{h}^2}\right]$$

Integrating we get,

$$\bar{w} = -\left[\left[\frac{\bar{z}^3}{6} - \frac{\bar{z}^2\bar{h}}{4}\right]\emptyset''(\bar{x}) - \frac{\bar{z}^2}{4}\frac{d\bar{h}}{d\bar{x}}\emptyset'(\bar{x}) - \frac{\bar{z}^2}{2\bar{h}^2}\frac{d\bar{h}}{d\bar{x}}\right] + c_3 \tag{14}$$

By imposing the boundary condition,

$$\bar{w} = 0 \; at \; \bar{z} = 0$$
$$\bar{w} = 0 \; at \; \bar{z} = \bar{h}(\bar{x})$$

we get $c_3 = 0$

Again,

$$0 = -\left[\left(\frac{\bar{h}^3(\bar{x})}{6} - \frac{\bar{h}^2(\bar{x})}{4}\bar{h}\right)\emptyset''(\bar{x}) - \frac{\bar{h}^2(\bar{x})}{4}\frac{d\bar{h}}{d\bar{x}}\emptyset'(\bar{x}) - \frac{\bar{h}^2(\bar{x})}{2\bar{h}^2}\frac{d\bar{h}}{d\bar{x}}\right] + 0$$

$$0 = -\left[-\frac{\bar{h}^3(\bar{x})}{12}\emptyset''(\bar{x}) - \frac{\bar{h}^2\bar{x}}{4}\frac{d\bar{h}}{d\bar{x}}\emptyset'(\bar{x}) - \frac{1}{2}\frac{d\bar{h}}{d\bar{x}}\right]$$

or, $$\frac{\bar{h}^3(\bar{x})}{12}\emptyset''(\bar{x}) + \frac{\bar{h}^2(\bar{x})}{4}\frac{d\bar{h}}{d\bar{x}}\emptyset'(\bar{x}) + \frac{1}{2}\frac{d\bar{h}}{d\bar{x}} = 0$$

This can be simplified as

$$\frac{d}{d\bar{x}}\left[\frac{\bar{z}^3(\bar{x})}{12}\emptyset'(\bar{x})\right]+\frac{1}{2}\frac{d\bar{h}}{d\bar{x}}=0$$

Assume a linear profile with $\bar{h}(\bar{x}) = k_1\bar{x} + k_2$ and further assume that $k_1 = k_2 = 1$. With this assumption, we get

$$\frac{d}{d\bar{x}}\left[\frac{(k_1\bar{x}+k_2)^2}{12}\emptyset'(\bar{x})\right]+\frac{1}{2}\frac{d}{d\bar{x}}(k_1\bar{x}+k_2)=0$$

Integrating first,

$$\frac{(k_1\bar{x}+k_2)^3}{12}\emptyset'(\bar{x})+\frac{1}{2}(k_1\bar{x}+k_2)=r_1$$

$$\emptyset'(\bar{x})=\frac{-\frac{1}{2}(k_1\bar{x}+k_2)+r_1}{\frac{(k_1\bar{x}+k_2)^3}{12}}$$

$$\emptyset'(\bar{x})=\frac{-6}{(k_1\bar{x}+k_2)^2}+\frac{r_1}{(k_1\bar{x}+k_2)^3}$$

Integrating we get,

$$\emptyset(x)=\frac{6k_1}{k_1\bar{x}+k_2}+\frac{k_1r_1}{(k_1\bar{x}+k_2)^2}+r_2$$

The constants of integration r_1 and r_2 are determined by boundary conditions on $\emptyset$ which are at $\emptyset = 0$ at $\bar{x} = 0$ and $\bar{x} = 1$.

$$r_1 + r_2 = -6$$

$$r_1 + 4r_2 = -12$$

$$\therefore\ r_1 = -4,\quad r_2 = -2$$

$$\emptyset(\bar{x})=\frac{6}{1+\bar{x}}+\frac{(-4)}{(1+\bar{x})^2}+(-2)$$

$$=\frac{6}{1+\bar{x}}-\frac{4}{(1+\bar{x})^2}-2$$

$$=\frac{2\bar{x}-2\bar{x}^2}{(1+\bar{x})^2}=\frac{2\bar{x}(1-\bar{x})}{(1+\bar{x})^2}$$

$$\therefore \bar{p}=\frac{2\bar{x}(1-\bar{x})}{(1+\bar{x})^2}$$

which is positive for $\bar{x} \in (0,1)$

Hence, pressure is developed inside the fluid and the bearing supports a load given by the integral of pressure between the limits $\bar{x} = 0$ and $\bar{x} = 1$

Hence,

$$\text{load } \emptyset=\int_0^1\frac{2\bar{x}(1-\bar{x})}{(1+\bar{x})^2}\,d\bar{x}$$

$$\text{load} = 6\,ln\,(2) - 4$$

Substituting the expression for pressure in equation (13) and (14), we get the expression of u and w respectively.

Having determined the velocities of the fluid, we now consider the viscous dissipation caused by motion. Assuming constant viscosity with respect to temperature, we have energy balance equation

$$\rho C_p \frac{DT}{Dt} = \frac{\partial Q}{\partial t} + \frac{\partial}{\partial x_j}\left(k \frac{\partial T}{\partial x_j}\right) + \emptyset$$

$$\text{where,} \quad \emptyset = \mu\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)\frac{\partial v_i}{\partial x_j}$$

Where ρ and k are density and thermal conductivity. The external energy q_i is neglected because it is not present.

Now, from energy Balance equation, we have

$$\rho c_{\mathbf{p}}\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + w\frac{\partial T}{\partial z}\right) = 2\mu\left[\frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] + k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}\right)$$

where $c_{\mathbf{p}}$ is the specific heat of the fluid and k is its thermal conductivity. Suppose that the plates are kept at a constant temperature T_1 and lubricant catches fire at critical temperature T_c. We proceed to non-dimensionalize this equation by scaling as before with

$$T = T_1 + \bar{\theta}(T_c - T_1)$$

Here $\bar{\theta}$ is the non-dimensional variable which varies between 0 and 1. The boundary conditions on $\bar{\theta}$ are given by,

$$\bar{\theta} = 0 \ at \ \bar{z} = 0, \bar{z} = \bar{h}(\bar{x})$$

Scaling and neglecting the terms that are small as compared to $\frac{1}{\epsilon}$, we get the simplified energy equation given by

$$\mu\frac{u_p^2}{l^2}\left(\frac{\partial \bar{u}}{\partial \bar{z}}\right)^2 = -\frac{k}{l^2}\frac{\partial^2 \bar{\theta}}{\partial \bar{z}^2}(T_c - T_1)$$

$$\left(\frac{\partial \bar{u}}{\partial \bar{z}}\right)^2 = -\frac{k}{\mu u_p^2}\frac{\partial^2 \bar{\theta}}{\partial \bar{z}^2}(T_c - T_1)$$

$$\left(\frac{\partial \bar{u}}{\partial \bar{z}}\right)^2 = -B\frac{\partial^2 \bar{\theta}}{\partial \bar{z}^2}$$

Where,

$$B = \frac{k}{\mu u_p^2}\left(T_c - T_1 \right)$$

Substituting the value of $\bar{u}$ from equation we get

$$\frac{\partial}{\partial \bar{z}}\left[\frac{(\bar{z}^2 - \bar{z}\bar{h})}{2}\frac{d\emptyset}{d\bar{x}} + \frac{\bar{z}}{\bar{h}}\right]^2 = -B\frac{\partial^2 \bar{\theta}}{\partial \bar{z}^2}$$

Integrating twice the equation, we get,

$$\frac{\bar{z}^4}{12}\emptyset'(\bar{x})^2 + \left[\frac{1}{\bar{h}} - \frac{\bar{h}}{2}\emptyset'(\bar{x})\right]^2\frac{\bar{z}^2}{2} + \frac{\bar{z}^3}{3}\emptyset'(\bar{x})\left[\frac{1}{\bar{h}} - \frac{\bar{h}}{2}\emptyset'(\bar{x})\right] + c_1\bar{z} + c_2 = -B\bar{\theta}$$

Using the boundary condition on $\bar{\theta}$, we get the expression for $\bar{\theta}$ as

$$-B\bar{\theta} = \emptyset'(\bar{x})^2\frac{(\bar{z}^4 - \bar{h}^3\bar{z})}{12} + \left[\frac{1}{\bar{h}} - \frac{\bar{h}}{2}\emptyset'(\bar{x})\right]^2\left(\frac{\bar{z}^2 - \bar{z}\bar{h}}{2}\right)$$
$$+ \emptyset'(\bar{x})\left[\frac{1}{\bar{h}} - \frac{\bar{h}}{2}\emptyset'(\bar{x})\right]\left(\frac{\bar{z}^3 - \bar{h}^2\bar{z}}{3}\right)$$

We note that $\bar{\theta}$ satisfies the boundary conditions at $\bar{z} = 0$ and $\bar{z} = \bar{h}(\bar{x})$ and for the case of linear profile, $\emptyset'(\bar{x})$ is

$$\emptyset'(\bar{x}) = -\frac{6k_1^2}{(k_1\bar{x} + k_2)^2} + \frac{8k_1^2}{(k_1\bar{x} + k_2)^3}$$

So $\bar{\theta}$ can be determined from equation

Here pressure is not zero and the bearing with non-parallel plates can support load easily.

Thus, the expression for temperature of a lubricant in slider bearing is derived. For the case when viscosity is constant, the expression is

$$B = \frac{k}{\mu u_p^2}\left(T_c - T_1 \right)$$

This gives conditions on possible values of various parameters of the bearing. However, when viscosity of the lubricant changes with temperature, due to nature of PDE involved, an explicit solution could not be derived.

3. Variable Viscosity

In the previous works, viscosity was considered constant. Here, variable viscosity is considered. For the liquid, the viscosity decreases with the temperature. From (3) and (4) we have

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} + \frac{1}{\rho}\frac{\partial p}{\partial x} = \frac{\partial}{\partial x}\left(v\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial z}\left(v\frac{\partial u}{\partial x}\right) \quad (15)$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z} + \frac{1}{\rho}\frac{\partial p}{\partial z} = \frac{\partial}{\partial x}\left(\frac{\partial w}{\partial x}\right) + \frac{\partial}{\partial z}\left(v\frac{\partial w}{\partial x}\right) \quad (16)$$

As in the previous work, the dimensional analysis give, where, $\nu = \nu_0 \bar{\nu}$

$$\frac{\partial \bar{p}}{\partial \bar{z}} = \frac{\partial}{\partial \bar{z}}\left(\nu \frac{\partial \bar{u}}{\partial \bar{z}}\right) \tag{17}$$

$$\frac{\partial \bar{p}}{\partial \bar{z}} = 0 \tag{18}$$

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{w}}{\partial \bar{z}} = 0$$

From (17), using $\nu = \nu_0 \bar{\nu}$

$$\frac{\partial \bar{p}}{\partial \bar{z}} = \nu_0 \frac{\partial}{\partial \bar{z}}\left(\bar{\nu} \frac{\partial \bar{u}}{\partial \bar{z}}\right) \text{ z}$$

Similarly, the energy equation reduces to

$$\bar{\nu} \left(\frac{\partial \bar{u}}{\partial \bar{z}}\right)^2 = -B \frac{\partial^2 \bar{\theta}}{\partial \bar{z}^2} \tag{19}$$

where,

$$B = \frac{k}{\nu_0 u_p^2} (T_c - T_1)$$

Let $\bar{\nu} = \frac{\alpha}{\bar{\theta}}$, as the viscosity changes with temperature, for the liquid.

From (18),

$$\bar{p} = \emptyset(\bar{x})$$

From (17)

$$\frac{\partial \bar{u}}{\partial \bar{x}} = \frac{(\emptyset'(x)\bar{z} + C_1(\bar{x}))}{\nu}$$

From (19),

$$\frac{\partial^2 \bar{\theta}}{\partial \bar{z}^2} = -\frac{\bar{\theta}}{\alpha B}\left(\emptyset'(x)\bar{z} + c_1(\bar{x})\right)^2$$

This equation is not easy to solve. If we can show that viscosity of the liquid changes with the increase in the temperature. Then the solution will have stability. This is the case of further investigation.

REFERENCES

[1] C. R. Doering, J. D. Gibbon; Applied Analysis of the Navier Stokes Equations; Cambridge University Press, 1995.

[2] D. P. Patel, Ishan Pendharkar, K. N. Uprety, Stephan Chapman, Alistair Fitt; UNESCO Regional International Workshop on Industrial Mathematics, IIT Bombay, 2-6 December 2006.

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