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Normality of the Hypersurface of Almost Hyperbolic Hermite Manifolds

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Hyperbolic Hermite manifold have been studied by Dube [2]. Hypersurfaces of almost hyperbolic Hermite manifold have been studied by Pal and Mishra [7] and Dube and Mishra [3] Bhatt [1] Dube [4]. The propose of the present paper is to study the normality of the hypersurfaces of almost hyperbolic Hermite manifolds.

1. Introduction :

An even dimensional differentiable manifold V_{2m}, on which there are defined tensor field F of type (1,1) and a metric tensor g, satisfying for arbitrary vector field $\lambda, \mu, \nu \ldots \in V_{2m}$

(1.1)a $$F^2 = I_{2m},$$

(1.1)b $$'F(\lambda,\mu) = g(\lambda,\mu) = -'F(\mu,\lambda)$$

is called an almost hyperbolic Hermite manifold with the almost hyperbolic Hermite structure $\{F,g\}$ [2]. Let V_{2m-1} be the hypersurface V_{2m} with the immersion map b and the corresponding Jacobian map B. Let h be the induced metric tensor and E be the induced Riemmian connection on V_{2m-1}, then we can write the arbitrary vector fields

$$X, Y, Z, \ldots \in V_{2m-1}.$$

(1.2)a $$D_{BX}BY = BE_X Y + 'H(X,Y)N,$$

(1.2)b $$D_{BX}N = -BHX$$

(1.2)c $$g(BX,BY)ob = h(X,Y),$$

where N is unit normal vector to V_{2m-1}, $'H$ is second fundamental tensor of V_{2m-1} and H is associate to $'H$ and D be the Riemannian Connexion on V_{2m},

(1.3) $$'H(X,Y) = h(HX,Y) = 'H(Y,X).$$

Let us write

(1.4)a $$FBX = BfX + U(X)N$$

(1.4)b $$FN = -B\,v$$

where f is a tensor of type (1,1) V is a vector field and U is a 1-form, then we have

(a) $f^2 - I_{2m-1} + U(X)V$

(b) $fV = 0$

(1.5) (c) $U o f = -1$

(d) $U(V) = -1$

(e) $'f(X,Y) = h(fX,Y) = -'f(Y,X)$,

An $(2m-1)$ dimensional differentiable manifold V_{2m-1} on which there are defined a tensor field f of type (1,1),.a vector field V, a 1-form U and a metric tensor field h, satisfying for arbitrary vector fields $X, Y, Z, \ldots \in V_{2m-1}$.

(1.5) a,c and (1.5)b,c is called an almost hyperbolic contact metric manifold with almost hyperbolic contact metric structure $\{f, u, v, h\}$ [3]. Thus from above we see that the hypersurface of an almost hyperbolic Hermite manifold is an almost hyperbolic contact metric manifold [3].

The author have proved the following :

The hyper-surface of a hyperbolic Kahler manifold is an almost hyperbolic contact metric manifold satisfying

(1.6)a $$(E_X f)Y = U(Y)HX - 'F(X,Y)V,$$

where

(1.6)b $$(E_X V) = fHX$$

(1.6)c $$(E_U f)Y = U(Y)HV - U(H,Y)V.$$

The equation of the hypersurface of a nearly hyperbolic Kaehler manifold is given by [1].

(1.7)a $$(E_X f)Y + (E_Y f)x = U(X)HY + U(Y)HX - 2'H(X,Y)V.$$

On this hypersurface we have

(1.7)b $$(E_V\, 'f)(Y,Z)U(Y)U(HZ) + U(Z)V(HY) = (E_Y U)(fZ) + 'H(Yf^2Z) = (E_Z U)(fY)$$

(1.7)c $$(E_X U)(Y) + (E_Y U)(X) = -'H(X,fY) - 'H(fX,Y)$$

The equation of the hypersurface of hyperbolic Hermite manifold is given by [1].

(1.8)a $$(E_{fX} f)fY + (E_X f)(Y) - U(Y)E_{fX}V + U(Y)(f^2Hf^2 + fHf)X + '\{H(fX,fY) - 'H(f2X,f2Y)\}V = 0.$$

Thus we have

(1.8)b $$(E_{fX}U)(fY) + (E^2_{fX}U)(Y) + 'H(fX,fY) - 'H(f^2X,f^2Y) = 0$$

The equation of hypersurface of a Quasi hyperbolic Kaehler manifold is given by [1]

(1.9) $$(E_{fX} f)fY + (E^2_{fX} f)Y - U(Y)E_{fX}V + (fHf - f^2Hf^2)X + \{'H(fX,fY) + 'H(f2X,f2Y)\} = 0.$$

On this hypersurface we have

(1.10) $$(E_{fX}U)fY - (E^2_{fX}U)Y + 'H((f^2X,fY) - 'H(fX,f^2Y) = 0$$

Definition : *An almost hyperbolic contact metric manifold is said to be normal if*

(1.10)a $$(E_{fX} f)fY - (E_X f)Y - U(Y)(E_{fX}V) = 0.$$

On this manifold we have

(1.10)b $$(E_{fX}U)(fY) = (E_X U)Y \Leftrightarrow (E_{fX}U)Y + (E_X U)FY = 0.$$

and $E_V f = 0$,

2. Some Results

Theorem 2.1. *If the hypersurface of hyperbolic kaehler manifold is normal, then we have*

$$fH = Hf \quad \text{and} \quad 'H(fX,Y) = -'H(X,fY)$$

Proof: From equation (1.10)b and (1.6)c, we have

(2.1)a $$U(Y)HV = U(HY)V \Rightarrow U(HY) = U(Y)U(HV)$$

(2.1)b $$HV = U(HV)Y$$

Substituting from equation (1.6)a,b in equation (1.10)a, we obtain

$$U(Y)\,(fHf + H)X + \{\,'H(fX,fY) - {}'H(X,Y)\}V = 0.$$

Now putting V for Y in the above equation, we get

$$FHf + Hf = U(HX)V,$$

which in a consequence of equation (2.1) yields

(2.2)a $$fHf + H - U(HV)\,U \otimes V = 0,$$

(2.2)b $$'H(fX,fY) = {}'H(X,Y) - U(X)U(Y)U(HV).$$

Equation (2.2)a and (2.2)b are equivalent to

(2.2)c $$fH = Hf$$

(2.2)d $$'H(fX,Y) = -{}'H(X,fY)$$

This completes the proof of the theorem.

Theorem 2.2. *If the hypersurface of nearly Hyperbolic Kaehler manifold is normal, then we have*

$$U(hfX) = 0$$

Proof: Replacing X and Y by fX and fY in equation (1.7)c we get

(2.3) $$(E_{fX}\,U)(fY) + (E_{fY}\,U)fX = -{}'H(fX, f^2Y) - {}'H(f^2X, fY).$$

Using equation (1.10)c in equation (1.7)c and equation (2.3) we get

(2.4)a $$'H(fX,Y) = 0.$$

Putting V for X in the above equation, we obtain

(2.4)b $$U(HfX) = 0.$$

Which gives the desired result.

Theorem 2.3. *If the hypersurface of nearly hyperbolic kaehler manifold is normal, it is the hypersurface of hyperbolic kaehler manifold.*

Proof: The equation (1.7)a of the hypersurface of a Nearly hyperbolic Kaehler manifold is equivalent to

(2.5) $$(E_X\,'f)\,(Y,Z) - U(Y)\,'H(X,Z) + U(Z)\,'H(X,Y) = (E_Z\,'f)\,(X,Y) - U(X)\,'H(Y,Z) + U(Y)\,'H(X,Z).$$

Substituting equation (2.5) in equation (1.10)a, we get

$$(2.6)\qquad -U(Z)\,'H(fX,fY)+(E_Z\,'f)(fX,fY)-(E_X\,'f)(Y,Z)$$
$$-U(Y)(E_{fX}U)(Z)=0.$$

On an almost hyperbolic Hermite manifold, we have

$$(2.7)\text{a}\qquad (E_Z\,'f)(fX,fY)+(E_X\,'f)(X,Y)=U(X)(E_Z U)(fY)$$
$$-U(Y)(E_Z U)f(X),$$

Using equation (2.5) in equations (2.6) and (2.7)a, we get

$$2(E_X\,'f)(Y,Z)-U(X)\{(E_Z U)(fY)-\,'H(Y,Z)\}=-U(Y)(E_Z U)(fX)$$
$$+(E_{fX}U)(Z)-2\,'H(X,Z)-U(Z)\{\,'H(fX,fY)+\,'H(X,Y)\}.$$

In view of equation (1.7)c, and equation (2.2)a, the above equation takes the form

$$(2.7)\text{b}\qquad 2(E_X\,'f)(Y,Z)-U(X)\{(E_Z U)(fY)-\,'H(Y,Z)\}-2U(Y)\,'H(X,Z)$$
$$-2U(Z)\,'H(X,Y)\}.$$

On hypersurfaces of nearly hyperbolic Kaehler manifold

$$(2.7)\text{c}\qquad 2(E_V\,'f)(Y,Z)-U(Y)(HZ)+U(Z)\,U(HY)=(E_Z V)\,fY+\,'H(Y,Z),$$

Now when the hypersurfaces is normal, we have $E_V f=0$.

Thus from equation (2.2)a and (2.7)c, we have

$$U(HX)=U(X)\,U(HV)\,;$$

Hence equation (2.7)b becomes

$$(2.7)\text{d}\qquad (E_X\,'f)(Y,Z)=U(Y)\,'H(X,Z)-U(Z)\,'H(X,Y)$$

which is the equation of hypersurfaces of hyperbolic Kaehler manifold.

This proves our assertions.

Theorem 2.4. *If the hypersurfaces of hyperbolic Hermite manifold is normal, then we have*

$$'H(fX,Y)-\,'H(X,fY)=0$$

Proof : Substituting from equation (1.10)a,b in (1.8)a, we get

$$(2.8)\qquad U(Y)\,(fHf+f^2Hf^2)X+\{\,'H(fX,fY)-\,'H(f^2X,f^2Y)\}=0.$$

The above equation holds, if

$$(2.8)\text{b}\qquad fHf+f^2Hf^2=0\Rightarrow\,'H(fX,fY)=\,'H(fX,fY)\,\,'H(f^2X,f^2Y),$$

which gives the required result

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