

NUMERICAL STUDY OF TRANSIENT BIO-HEAT TRANSFER MODEL  
WITH HEAT TRANSFER COEFFICIENT AND CONDUCTION EFFECT  
IN CYLINDRICAL LIVING TISSUE

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**Abstract:** The human thermal comfort is affected by the body's heat exchange mechanism conduction, convection, radiation, and evaporation. The mode of heat transfer between the body and environment depends upon the human internal physiological phenomena, together with the boundary conditions. The present paper provides the comprehensive overview of the thermoregulatory system of human body and studies the numerical solution of unsteady-state one dimensional Pennes bio-heat equation with appropriate boundary conditions. The solution is used to observe the temperature profiles at different thermal conductivities, and different heat transfer coefficients in the living tissue at the various time steps. Various physical and physiological factors across the cylindrical living tissue have been incorporated in the model.

**Key Words:** Thermoregulatory control, Heat exchange mechanism, Unsteady - state, Cylindrical living Tissue

**AMS (MOS) Subject Classification.** 92C35 80A20

## 1. INTRODUCTION

Human body has the complex vascular geometry involving the multiple physical and physiological phenomena such as conduction, convection, radiation, sweat evaporation, blood flow and metabolism. Heat produced by human body may either preserved or transmitted to the environment. When the internal body core temperature is nearly  $37^{\circ}C$ , human feels better comfort. So this temperature is considered as the normal temperature which is as the result of heat generation and the heat loss by the body[11]. According to Report of WHO, published in 1969, It is not recommended that body core temperature exceeds  $38^{\circ}C$  for a daily exposure to heavy work." The fluctuation in this uniform body temperature so far above and below causes the disturbance in thermoregulatory system. So one should always try to keep balance the body temperature around  $37^{\circ}C$  within the range  $\pm 0.6^{\circ}C$ .

Thermoregulation is the process controlling the internal body temperature through the hypothalamus heat production and heat loss center. The body also uses other processes like dilating or constricting blood vessels, sweating and shivering.

Metabolism, the major source of heat generation which differs from layer to layer with the highest heat generation in brain almost  $13400 w/m^3$  and in contrast, no heat production in bone  $0 w/m^3$ [23]. In this study heat is assumed to be generated uniformly by metabolic and chemical reactions through the cylindrical living tissue. On the other hand, arterial and venous including the small blood vessels located in tissues also play a vital role for heat transfer between blood capillaries, and the tissues. The assumption and calculation made here is based on the Fick's law of perfusion. These important components of the heat balance under the transient condition, the thermal energy generated or transferred to the body part may go to alter the amount storage inside it.

The physical process of several mechanisms such as conduction, convection, sweat evaporation and radiation are the causes for the heat loss from the body. Though convection is the major part for heat loss from the body, in hotter environment heat loss depends more on evaporation. The heat exchange between skin surface and environment is determined by the amount of body area expose to outer environment [4, 20]. Evaluation of these physical and physiological parameters is a major task for the analysis of the heat transfer, and the thermoregulatory control.

Sweating is an essential process to regulate homeostasis in the human body. The brain, and body work within a delicate balance to ensure that the person's temperature is neither too high nor too low. At a constant core temperature, the sweating rate is proportional to the skin temperature and vice versa [20]. The weighted mean value of body, and skin temperature is taken to calculate sweat rate which is given by the valid equation [11].

$$E = [8.47 \times 10^{-5}(0.1 \times T_s + 0.9 \times T_a) - 36.6^0C] (kg/m^2.s)$$

where,  $T_s$  = skin surface temperature, and  $T_a$  = body core temperature.

Since last few decades the study of bioheat transfer problems became emerging area for research. Various models related to the heat transfer in biological tissue using Pennes' bioheat equations are handled by several researchers after the Pennes' model in 1948 [9]. Gurung and Saxena [5], have used the Finite Element Approach to investigate the one dimensional steady-state temperature distribution in the dermal parts with quadratic shape function. Saxena and Bindra [21] have used Pseudo-Analytical Finite Partition Approach to the temperature distribution problem. In [20], Gurung and Acharya have simulated numerically the sex-related differences in the sensitivity of the sweating heat response to change in body temperature. Khandey and Hussian [14] have investigated about the human peripheral tissue temperature during exposure to serve cold stress. They have used explicit formula of finite difference method for simulation. Gurung and Saxena [4] have studied about the transient temperature distribution in human dermal part with protective layer at low atmospheric temperature. Recently Roohi et al. [19] have developed the comprehensive model for the numerical study of space-time fractional bioheat equation. They have used

fractional-order Legendre function in their study.

As human body has the complex vascular distribution pattern embedded inside the tissue, the study of heat transfer in such living biological tissue is really a cumbersome phenomena. Bioheat transfer processes in living tissues are affected by various physical and physiological parameters, surrounding environments, initial and boundary conditions along with temperature-dependent metabolic heat generation [20].

As the body temperature may fall or rise according to the changes in external environment and other above mentioned physical and physiological parameters, one can be minimize and keep the temperature balance by engaging himself in exercise during cold and sweating during hot environment[6]. The heat balance given in the relation is

$$\text{Heat Store} = \text{Heat Production} - \text{Heat Loss}$$

Here,

$$\text{Heat Production} : \text{Metabolic Heat Generation,}$$

and

$$\begin{aligned} \text{Heat Loss:} & \text{Conduction} + \text{convection} + \text{Radiation} + \text{Evaporation} \\ & + \text{Evaporation} + \text{Respiration} \end{aligned}$$

Negative heat storage shows the more heat loss than production and in this case body starts cooling whereas positive heat storage shows the metabolic rate is higher than the sum of all heat losses and the body temperature rises.

The transient temperature profiles in the human body may helpful for the medical persons who monitor the temperature fluctuations in the tissue during the hyperthermia treatment against cancer .

The present paper focuses the study of transient solution of one dimensional bioheat transfer model and apply it to estimate the effect of higher and lower thermal conductivities in cylindrical living tissue. The model, Pennes' bioheat equation is solved by using finite difference technique with appropriate boundary conditions at the various time steps. Temperature profiles at various heat transfer coefficients and the metabolic heat generations have also been observed.

## 2. MODEL FOR HEAT TRANSFER

One dimensional time dependent governing differential equation is used as the basic mathematical model for the heat transfer which is given by

$$(2.1) \quad \rho c \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} + W_b c_b (T_a - T) + q_m$$

This bioheat equation (2.1) is suggested by H. Pennes' in 1948. The left hand side is the total heat storage; and the first and second terms of right hand side are, respectively guided by Fick's laws of diffusion and perfusion whereas the third term is the rate of metabolic heat generation.

As the recent paper aims the study of temperature profiles in cylindrical shape of the human body. The cylindrical form of this bio heat equation in radial direction is performed here.

$$(2.2) \quad \rho c \frac{\partial T}{\partial t} = k \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) \right] + W_b c_b (T_a - T) + q_m$$

Where,  $\rho$ : tissue density ( $kg/m^3$ ),  $c$ : tissue specific heat ( $j/kg^0C$ )  
 $k$ : thermal conductivity ( $w/m^0C$ ),  $W_b$ : blood perfusion rate ( $kg/m^3.s$ )  
 $c_b$ : tissue specific heat ( $j/kg^0C$ ),  $T_a$ : arterial blood temperature ( $^0C$ ).  
 $q_m$ : metabolic heat generation ( $w/m^3$ ),  $r$ : radial distance from centre of core towards skin surface (m)

**2.1. Boundary Conditions:** The inner boundary condition of the living tissue is considered uniform and taken as;

$$(2.3) \quad \text{at } r = 0, \quad \frac{\partial T}{\partial r} = 0$$

There is continuous heat flux between the skin surface and atmospheric environment as outer surface of skin is exposed to external environment [17]. In this case heat loss from the body is caused by convection, radiation and evaporation. The Robin boundary condition guided by Newton's law of cooling is given by

$$(2.4) \quad \text{at } r = R, \quad -k \frac{\partial T}{\partial r} = h_c (T - T_\infty) + LE$$

Where,  $h_c$ : combined heat transfer coefficient due to convection and radiation  
 $L$ : latent heat,  $E$ : sweat evaporation,  $T_\infty$ : Environmental temperature

**2.2. Initial Condition:** For the time dependent boundary value problem, the initial condition is given by

$$(2.5) \quad T(r, 0) = T_0(r)$$

### 3. FINITE DIFFERENCE SCHEME FOR SOLUTION OF THE MODEL

One dimensional form of cylindrical tissue is divided into  $R+1$  discrete points uniquely specified by spatial indices,  $r_i = i\Delta r'$  in the radial direction. The discretization of circular cross section of peripheral human limb where the temperature flow in axial direction is uniform as shown in figure 1

In the time discretization,  $\Delta t$  is denoted by the discrete time step size, and the total time to evaluate the temperature is  $t^n = n\Delta t$ .

In finite difference scheme the differential equation with continuous derivative is approximately expressed in the system of difference equation by using Taylor's series expansion. Writing equation (2.2) by using implicit finite difference scheme for RHS terms, and forward

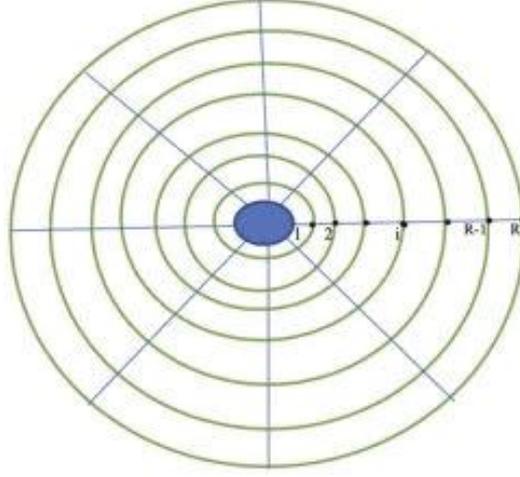


FIGURE 1. Circular cross section of peripheral human limb

difference for the term in left side we get,

$$(3.1) \quad \frac{\rho c}{\Delta t} [T_i^{n+1} - T_i^n] = k \left[ \frac{T_{i-1}^{n+1} - 2T_i^{n+1} + T_{i+1}^{n+1}}{(\Delta r)^2} \right] + \frac{k}{r_i} \left[ \frac{T_{i+1}^{n+1} - T_{i-1}^{n+1}}{2\Delta r} \right] + W_b c_b (T_a - T_i^{n+1}) + q_m \quad i = 1, 2, \dots, R-1$$

$$\text{For } D = \frac{k}{\rho c}, \lambda = \frac{D\Delta t}{\Delta r^2}, \mu = \frac{D\Delta t}{\Delta r}, M = \frac{W_b c_b}{\rho c}, S = \frac{q_m}{\rho c}, F = \Delta t (MT_a + S)$$

we have,

$$(3.2) \quad \left( -\lambda + \frac{\mu}{2r_i} \right) T_{i-1}^{n+1} + (1 + 2\lambda + M) T_i^{n+1} + \left( -\lambda - \frac{\mu}{2r_i} \right) T_{i+1}^{n+1} - F = T_i^n$$

$$D_i T_{i-1}^{n+1} + E_i T_i^{n+1} + B_i T_{i+1}^{n+1} - F = T_i^n$$

with  $i = 1, 2, \dots, R-1$

where,  $E_i = (1 + 2\lambda + M)$ ,  $D_i = \left( -\lambda + \frac{\mu}{2r_i} \right)$  and  $B_i = \left( -\lambda - \frac{\mu}{2r_i} \right)$   
for,  $i = 1, 2, \dots, R$

The equation (3.2) is Finite difference scheme for interior nodes of the equation (2.2).

**3.1. FD Scheme at Boundary  $r = 0$ :** The cylindrical thickness  $r$  is measured from body core as shown in figure 2. At the body core, both  $r$  and the heat flux  $\frac{\partial T}{\partial r}$ , are zero, then  $\frac{1}{r} \left( \frac{\partial T}{\partial r} \right)$  approaches to indeterminate form  $\frac{0}{0}$  as  $r \rightarrow 0$ .

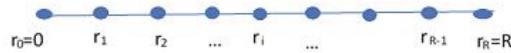


FIGURE 2. Discretization in radial direction

The use of L'Hospital rule, then gives

$$\frac{1}{r} \frac{\partial T}{\partial r} \Big|_{r=0} = \frac{(\frac{\partial}{\partial r})(\frac{\partial T}{\partial r})}{\frac{\partial}{\partial r}(r)} = \frac{\partial^2 T}{\partial r^2} \Big|_{r=0}$$

Now equation (2.2) becomes,

$$(3.3) \quad \frac{\partial T}{\partial t} = \frac{2k}{\rho c} \left( \frac{\partial^2 T}{\partial r^2} \right) + \frac{W_b c_b}{\rho c} (T_a - T) + \frac{q_m}{\rho c}$$

The finite different scheme of equation (3.3) at  $r = 0$  is

$$(3.4) \quad -2\lambda T_{-1}^{n+1} + (1 + 4\lambda + M) T_0^{n+1} - 2\lambda T_1^{n+1} - F = T_0^n$$

The FD scheme for  $\frac{\partial T}{\partial r} = 0$  is

$$(3.5) \quad T_{-1}^{n+1} = T_1^{n+1}$$

Using equation (3.5) in equation (3.4), we obtain

$$(3.6) \quad E_0 T_0^{n+1} - 4\lambda T_1^{n+1} - F = T_0^n$$

where,  $E_0 = (1 + 4\lambda + M)$

**3.2. FD Scheme at Boundary  $r = R$ :** The central difference approximation is,

$$(3.7) \quad T_{R+1}^{n+1} = T_{R-1}^{n+1} - \frac{2\Delta r h_c}{k} (T_R^{n+1} - T_\infty) - \frac{2\Delta r L E}{k}$$

Then FD equation at  $r = R$  of equation (2.2) is

$$(3.8) \quad -2\lambda T_{R-1}^{n+1} + (E_R - 2\Delta r h_c B_R / k) T_R^{n+1} + F_R - F = T_R^n$$

where,  $F_R = \frac{2\Delta r B_R}{k} (h_c T_\infty - L E)$

Writing the equations (3.6), (3.2), and (3.8) in the matrix equation form

$$(3.9) \quad A T^{n+1} = T^n + B$$

where,  $T^n = [T_0^n \quad T_1^n \quad T_2^n \quad \dots \quad T_r^n]^T$

$$A = \begin{bmatrix} E_0 & -4\lambda & 0 & 0 & \dots & 0 \\ D_1 & E_1 & B_1 & 0 & \dots & 0 \\ 0 & D_2 & E_2 & B_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \dots & \dots & -2\lambda & (E_R - B_R \frac{2h_c \Delta r}{k}) \end{bmatrix}$$

$$T^{n+1} = \begin{bmatrix} T_0^{n+1} \\ T_1^{n+1} \\ T_2^{n+1} \\ \vdots \\ T_R^{n+1} \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} F \\ F \\ F \\ \vdots \\ F \\ F - F_R \end{bmatrix}$$

4. RESULTS AND DISCUSSION

The heat transfer model in living tissue depends upon the various biological properties as well as thermophysical parameters. In this study the cylindrical limb is uniformly discretized into the number of nodes in the radial direction where the heat flow is started from the core of the body towards skin surface as already shown in the figure 2.

The effect of various values of heat transfer coefficients and thermal conductivities are shown in figures 3 and 4 respectively. The graphs in these figures are obtained by using the computer software Python.

**4.1. Effect of Heat Transfer Coefficient:** Temperature profiles in the case of a high and low heat transfer coefficients has been observed at the different time steps. The size of space domain (tissue thickness)  $R$  has been taken 0.03 m. In this case, the values of parameters have been assigned as follows [11].

$$k = 0.48 \text{ w/m}^0\text{C}, c_b = 1000 \text{ j/kg}^0\text{C}, W_b = 3.5 \text{ kg/m}^3\text{.s}, T_a = 37^0\text{C},$$

$$L = 24 \times 10^5 \text{ j/kg}, E = 4 \times 10^{-5} \text{ kg/m}^2\text{.s}, \text{ and } T_\infty = 22^0\text{C}.$$

The system of equation (3.9) with these parametric values gives the graphs in figure 3(a), and figure 3(b) for the time dependent temperature profiles when the heat transfer coefficients  $h_c$  are  $10.023 \text{ w/m}^2\text{.}^0\text{C}$  and  $30.23 \text{ w/m}^2\text{.}^0\text{C}$  respectively [11]. The nude human body

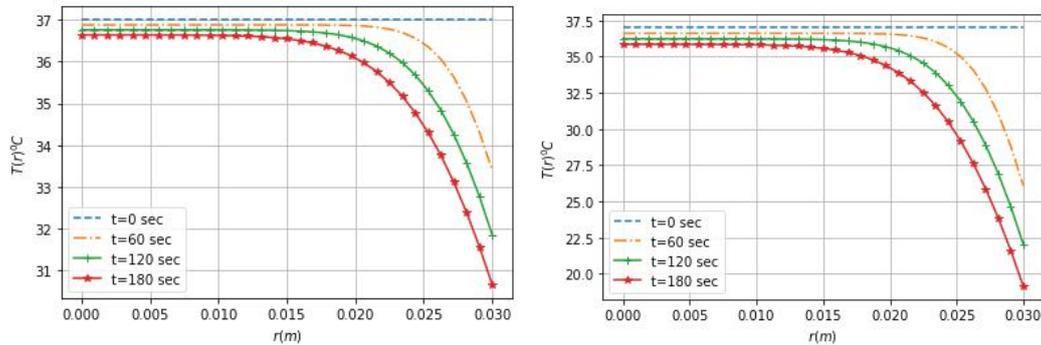


FIGURE 3. Radial Temperature profile at(a)  $h_c = 10.023 \text{ w/m}^2\text{.}^0\text{C}$  (b)  $h_c = 30.023 \text{ w/m}^2\text{.}^0\text{C}$

surface is directly affected by the outer environmental condition where the convection and radiation heat transfer coefficient appears. Figure 3(a) represents the temperature profiles at rest, 60, 120, and 180 seconds. Except rest ( $t = 0$ ) the temperature in skin from the body core with certain radial distance is uniform, i.e. steady state and then it goes down slowly towards the skin surface. The temperature at skin surface is  $33.5^0\text{C}$  in 60 second,  $32^0\text{C}$  in 120 second, and  $29^0\text{C}$  in 180 second. On the other hand, in figure 3(b), the temperature in the skin from body core from the core of body towards the skin surface slows sharply down. In 60 second, the temperature reaches  $26^0\text{C}$ , in 120 second  $22^0\text{C}$ , and in 180 second it reaches to  $18^0\text{C}$ . This is due to higher heat transfer coefficient. Thus higher heat transfer coefficient has more capacity to reduce the body surface temperature than that of the lower heat transfer coefficient.

**4.2. Effect of Thermal Conductivities:** Heat loss through the lower and higher thermal conductivities have been calculated by assigning the following parametric values [11].

$$h_c = 20.023 \text{ w/m}^2 \cdot ^\circ\text{C}, T_\infty = 22^\circ\text{C}, c_b = 1000 \text{ j/kg}^\circ\text{C}, W_b = 3.5 \text{ kg/s.m}^3, \\ T_a = 37^\circ\text{C}, L = 24 \times 10^5 \text{ j/kg}, \text{ and } E = 4 \times 10^{-5} \text{ kg/m}^2 \cdot \text{s},$$

The system of equation (3.9) gives the graph in figure 4(a) and figure 4(b) for time dependent temperature profiles at the values of thermal conductivities  $k$  are  $0.24 \text{ w/m}^\circ\text{C}$  and  $0.72 \text{ w/m}^\circ\text{C}$  respectively [11].

From Figure 4(a), we observe that except rest, the temperature in the skin from body

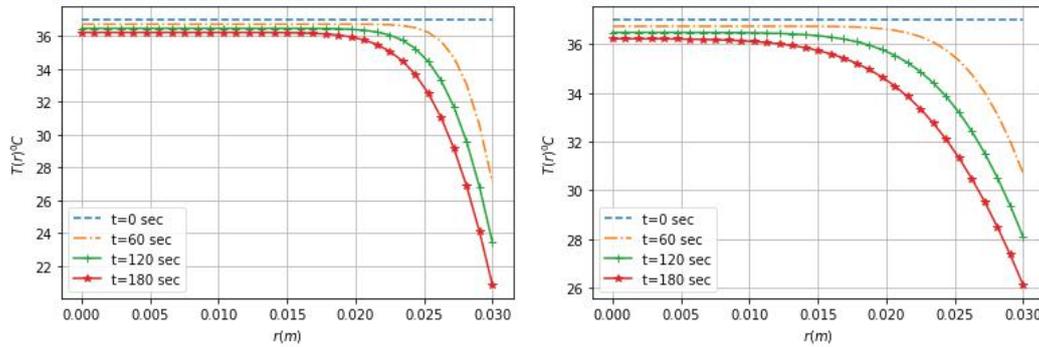


FIGURE 4. Radial Temperature profile at (a)  $k = 0.24 \text{ w/m}^\circ\text{C}$  (b)  $k = 0.72 \text{ w/m}^\circ\text{C}$

core upto certain radial distance is uniform as in the case of heat transfer coefficients, i.e. steady state. After then the temperature decreases rapidly towards the skin surface and the temperature at skin surface is  $27^\circ\text{C}$  in 60 second,  $23^\circ\text{C}$  in 120 second, and  $21^\circ\text{C}$  in 180 second. On the other hand, in figure 4(b), it is found that the temperature profile in the skin from the body core towards the skin surface decreases smoothly down. In 60 second, the temperature reaches  $31^\circ\text{C}$ , in 120 second  $28^\circ\text{C}$ , and in 180 second it reaches up to  $26^\circ\text{C}$ . Thus higher thermal conductivities causes to rise in the skin surface temperature than the lower thermal conductivity.

## 5. CONCLUSION

A time dependent bioheat transfer model is solved using the implicit finite difference method for analyzing the heat transfer coefficient and conduction effect in the cylindrical shape of human body. The result shows that the temperature at the skin surface decreases significantly on the increase of heat transfer coefficient in the different time steps. While increasing in thermal conductivity the temperature on the skin surface increases. On one hand the graphs show the role of convection and conduction for heat loss from skin surface, on the other hand, this paper provides the knowledge of prevention of physiological disturbance due to several phenomena and the important and comprehensive overview of the thermoregulatory system of human body. There is inverse relation in time and temperature if  $T_\infty < 37^\circ\text{C}$ . This paper is bound to be helpful for those who involve themselves

in the medical field, such as hyperthermia treatment against cancer and the biomedical researcher for further investigation in thermal disturbance. This paper can be extended in annular, and axial direction by incorporating the clothing effect in thermoregulatory system.

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