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On certain Köthe-Toeplitz duals

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Abstract In this chapter we deal on the results corresponding to Dutta, Srivastava, Gurusingh and others [6, 7]. Promising from this and based on these results we introduce a general sequence space X_t where X is any sequence space. We establish some inclusion relations, topological results and characterize $\alpha-$, $\beta-$ and $\gamma-$ duals of X_t in terms of the $\alpha-$, $\beta-$ and $\gamma-$ duals of X. Furthermore we characterize the Köthe Toeplitz duals and discuss the perfectness of these sequence spaces.

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1. Introduction

The following definitions and notations will be useful in our discussion and presentation.

$\ell_\infty =$ The space of all bounded sequences

$$= \left\{ x = (x_k) : \sup_k |x_x| < \infty \right\},$$

$c =$ The space of all convergent sequences

$$= \left\{ x = (x_k) : \left| x_k - \ell \right| \to 0 \text{ for some } \ell \varepsilon C \right\},$$

$c_0 =$ The space of all null sequences

$= \left\{x = (x_k) : |x_k| \to 0 (k \to \infty)\right\}$, and

$\ell_p = (1 \le p < \infty)$. The space of sequences $x = (x_k)$ with absolutely

$p =$ summable series.

If $p = (p_k)$ is a bounded sequence of strictly positive real numbers then;

$$\ell_\infty(p) = \left\{x = (x_k) : \sup_k |x_x|^{p_k} < \infty\right\},$$

$$= \left\{x = (x_k) : |x_k|^{p_k} \to 0 \text{ as } k \to \infty\right\},$$

$$c(p) = \left\{x = (x_k) : |x_k - \ell|^{p_k} \to 0 \text{ for some } k \to C\right\},$$

$$\ell(p) = \left\{x = (x_k) : \sum_k |x_k|^{p_k} < \infty\right\},$$

$$c = \left\{x = \{x_k\} : |x_k| \to \ell (k \to \infty), \text{ for some } \ell \varepsilon C\right\}, \text{ and}$$

and $$c_s(p) = \left\{x = (x_k) = \frac{1}{n} \sum_{k=1}^{n} |x_k - \ell|^{p_k} \to 0 \text{ for some } \ell \varepsilon C\right\}.$$

Let $t = (t_k)$ be any fixed sequence of nonzero complex numbers satisfying

$$\mathrm{Lim}_k \inf (t_k)^{1/k} = r (0 < r \le \infty)$$

and let X be any sequence space. Then we define X_t by

$$X_t = \{x = (x_k) : (t_k x_k) \varepsilon X\}$$

For detailed discussion on these spaces we refer [1, 2, 3, 4, 5, 9, 10,12] to the reader. In this chapter we give some topological relations between X and (X_t), and also we give the $\alpha-$, $\beta-$ and $\gamma-$duals of X_t in terms of the $\alpha-$, β and $\gamma-$ duals of X.

2 Some Topological Properties of X_t

In this section we give some topological relations between X_t and X, and we discuss some properties of X_t.

Theorem 2.1 *If X is a complete paranormed space, then X_t is also a complete paranormed space.*

Proof: Since $(0)\varepsilon X_t, X_t \neq 0$. It is easy to check that X_t is a linear space. And also it is clear that the function g^* is defined by

$$g^*(x) = g(tx)$$

where g is the paranorm in X, is a paranorm because $g^*(0) = 0$, $g^*(x) = g^*(-x)$ and $g^*(x+y) \leq g^*(x) + g^*(y)$. Now clearly $\lambda_n \to \lambda$ in C and $g^*(x-x) \to 0$ as $n \to \infty$ imply that $g^*(\lambda_n x^n - \lambda x) \to 0$ as $(n \to \infty)$, where $x^n = (x_k^n)_k = (x_1^n, x_2^n, x_3^n, \ldots, x_k^n, \ldots)$ and $x = (x_k)$.

To show that X_t is complete, let (x^n) be a Cauchy sequence in X_t, where $x^n = (x_1^n, x_2^n, \ldots)\varepsilon X_t$. Then $(tx^n) = \left((t_k x_k^1), (t_k x_k^2), \ldots\right)$ is a Cauchy sequence in X. Since X is complete, it converges to (z_k) say. Let $z_k = t_k z_k$, so that $x_k = t_k^{-1} z_k$. Then (tx^n) converges to $(t_k x_k)$ in X.

Hence,

$$g(t_k x_k^n) = (t_k x_k) = g\left(t(x^n - x)\right) \to 0$$

$$\left(g(t_k x_k^n) - (t_k x_k)\right) = g\left(t(x^n - x)\right) \to 0) \text{ as } n \to \infty, \text{ which implies that}$$

$$g^*(x^n - x) \to 0 \text{ as } n \to \infty$$

Therefore x^n is convergent, consequently X_t is a complete paranormed space. This completes the proof of the theorem.

Corollary 2.1: *If X is a Banach space, then so is X_t. Here the norm in X_t is defined by $\| x \|_{(t)} = \| (t_k x_k) \|$, where $\| \;\; \|$ is the norm in X.*

Corollary 2.1

(i) $(\ell_\infty)_t (c_0)_t (c)_t$ are BK space with the norm $\| x \|_t = \| t_k x_k \|$

Lemma 2.1 If $X \subset Y$ then $X_t \subset Y_t$ and $X_t^* \subset Y_t^*$, where $X_t^* = \{x = (x_k) : (x_k t_k^{-1})\varepsilon X\}$,

(i) $(\cup_i X_i)_t = \cup_i (X_i)_t$

(ii) $(\cap_i Y_i)_t = \cap_i (Y_i)_t$

The proof of this Lemma 1 is easy and hence omitted.

Theorem 2.2 *Let X be a complete paranormed space and let Z be a closed subset of X then Z_t is a closed subset of X_t.*

Proof: Since $Z \subset X$, $Z_t \subset X_t$ by Lemma-1. Now let $x \varepsilon (\overline{Z_t})$, then there exists a sequence $(x^n) \subset Z_t$ such that (x^n) converges to x. This implies that $g^*(x^n - x) = g^*((t_k x_k)^n - (x_k)) \to 0$ as $n \to \infty$ in Z_t. Thus $g((t_k x_k^n) - (t_k x_k)) \to 0$ as $n \to \infty$ in Z. Hence $(t_k x_k)$ is the limit of a sequence of points in Z.
Therefore $(t_k x_k) \varepsilon \overline{Z}$ which gives that $x \varepsilon (\overline{Z_t})$. Conversely if $x \varepsilon (\overline{Z})_t$ then $x \varepsilon (\overline{Z})_t$, since Z is closed that is $\overline{Z} = Z$. Therefore $(\overline{Z}_t) = (\overline{Z})_t = Z_t$, hence Z_t is closed in X_t.
This completes the proof of the theorem.

Corollary 2.3: *Let X be a Banach space and Z be a closed subset of X. Then Z_t is a closed subset of X_t.*

Theorem 2.3 *If X is a separable space, so is X_t.*

Proof: Let X be a separable space. Then there exists a countable subset Z of X such that $\overline{Z} = X$. Then $(\overline{Z}_t) = X_t$ by Theorem 4.2.2. Hence Z_t is dense in X_t. Let us define $f : Z_t \to Z$ by $f(x) = (t_k x_k)$. It is clear that f is bijective. Since Z is countable, Z_t is also a countable subset of X_t. Hence X_t is separable.
This completes the proof of the theorem.

Theorem 2.4 *If X is a Hilbert space then X_t is also a Hilbert space.*

Proof: Let X be a Hilbert space. If we define the inner product $\langle\ \rangle_t$ in X_t by

$$\langle x, y \rangle_t = \langle (t_k x_k), (t_k y_k) \rangle, \quad (x, y \varepsilon X_t)$$

Where $\langle\ \rangle$ denotes the inner product in X. It is easily seen that $\langle\ \rangle_t$ satisfies the conditions of inner product, so X_t is an inner product space and hence X_t is a Hilbert space.

Remarks: X_t need not to be a sequence algebra even if X is so. Indeed, it is known that c_0 is a sequence algebra. But $(c_0)_t$ is not a sequence algebra for $(t_k) = (1/k)$. For let $x = (\sqrt{k})$ and $y = (\lambda\sqrt{k})$, where $\lambda \varepsilon C$ is a constant. Then $x, y \varepsilon (c_0)_t$ but $z \notin (c_0)_t$, where $z = (x_k y_k)$.

3 Köthe Toeplitz Duals of X_t

In this section first we give the α-duals of $\ell_\infty(p), c_0(p), c(p)$ and $\ell(p)$, and we discuss the second duals and perfectness of some sequence spaces. Then we characterize the $\alpha-$, $\beta-$ and $\gamma-$ duals of X_t in terms of the $\alpha-$, $\beta-$ and $\gamma-$ duals of X respectively.

We also discuss the second duals and perfectness of X_t for various sequence spaces X. It may be noted here that $\beta-$duals of $\ell_\infty(p), c_0(p), c(p)$ and $\ell(p)$ have been characterized in [6,7].

Definition 3.1 Let X be a sequence space and define

(i) $X^\alpha = \left\{ a = (a_k) : \sum_k |a_k x_k| < \infty \text{ for all } x \,\varepsilon\, X \right\}$,

(ii) $X^\beta = \left\{ a = (a_k) : \sum_k a_k x_k \text{ converges for all } x \,\varepsilon\, X \right\}$,

(iii) $X^\gamma = \{a = (a_k) : \sup_n \left| \sum_{k=1}^{\infty} a_k x_k \right| < \infty \text{ for all } x \,\varepsilon\, X\}$

Then X^α, X^β, and X^γ are called the $\alpha-, \beta-$and $\gamma-$dual spaces of X respectively. X^α is also called Köthe- Toeplitz dual space and X^β is called the generalized Köthe-Toeplitz dual space. It is easy to show that $\phi \subset X^\alpha \subset X^\beta \subset X^\gamma$. If $X \subset Y$, then $Y^\eta \subset X^\eta$ for $\eta = \alpha, \beta$ or γ. Also for a sequence space X it is clear that $X \subset (X^\eta)^\eta = X^{\eta\eta}$, where $\eta = \alpha, \beta$ or γ.

Definition 3.2 For a sequence space X, if $X = X^{\eta\eta}$, then X is called an η-space, where $\eta = \alpha, \beta$ or γ. In particular, an α-space is called Köthe space or a perfect sequence space.

Theorem 3.1 *Let η denote α, β or γ. Then*

(i) If the η-dual X^η exists, then $(X_t)^\eta$ exists and

$$(X_t)^\eta = \left\{ a = \langle a_k \rangle : \left\langle \frac{a_k}{t_k} \right\rangle \varepsilon\, X^\eta \right\} = \langle X^\eta \rangle_t$$

(ii) If $X^{\eta\eta}$ exists, then $(X_t)^{\eta\eta}$ exists and

$$(X_t)^{\eta\eta} = \left\{ x = (x_k) : (t_k x_k) \,\varepsilon\, X^{\eta\eta} \right\} = (X^{\eta\eta})_t$$

Proof: Let $\eta = \alpha$ and $D = \left\{ a = (a_k) : (a_k / t_k) \,\varepsilon\, X^\alpha \right\}$. We show that $(X_t)^\alpha = D$. Let $a \,\varepsilon\, (X_t)^\alpha$ then $\Sigma_k |a_k x_k| < \infty$ for every $x \,\varepsilon\, X_t$ so that

$$\sum_k \left| \frac{a_k}{t_k} t_k x_k \right| = \sum_k |a_k x_k| < \infty .$$

Since $(t_k x_k) \,\varepsilon\, X$ it follows that $(a_k / t_k) \,\varepsilon\, X^\alpha$ which implies that $a \,\varepsilon\, D$. Hence

$$(X_t)^\alpha \subset D.$$

Conversely, if $a \varepsilon D$ and $x \varepsilon X_t$, then $(a_k / t_k) \varepsilon X^\alpha$ and $(t_k x_k) \varepsilon X$ so that

$$\sum_k |a_k x_k| = \sum_k \left| \frac{a_k}{t_k} t_k x_k \right| < \infty$$

As $a \varepsilon X_t$ it follows that $a \varepsilon (X_t)^\alpha$. Hence $D \varepsilon (X_t)^\alpha$.

Consequently $(X_t)^\alpha = (X^\alpha)_t$.

For $\eta = \beta$ and $\eta = \gamma$ the proofs are similar. Therefore we omit them.

(i) Let $\eta = \alpha$ and let $X^{\alpha\alpha}$ exist. Then

$$\begin{aligned}(X_t)^{\alpha\alpha} &= [(X_t)^\alpha]^\alpha \\ &= [(X^\alpha)_t]^\alpha \\ &= (X^{\alpha\alpha})_t\end{aligned}$$

For $\eta = \beta$ and $\eta = \gamma$ the proof follows as in (i).

This completes the proof of the theorem.

Theorem 3.2 *X_t is an η-space if and only if X is an η-space, where $\eta = \alpha, \beta$ or γ.*

Proof: Let X be an η-space. Then $X^{\alpha\alpha} = X$. Now $(X_t)^{\alpha\alpha} = (X^{\alpha\alpha})_t$ by Theorem 4.3.1 and hence $(X_t)^{\eta\eta} = X_t$. Thus X_t is an η-space.

Conversely, if X_t is an η-space then $(X_t)^{\eta\eta} = X_t$ which implies that $(X^{\eta\eta})_t = X_t$ by Theorem 3.1 (ii). From Lemma (1) it follows that $X^{\eta\eta} = X$ that is, X is an η-space.

Theorem 3.3. *Let $0 < p_k \leq 1$ for every k. Then the following statements are equivalent:*

(i) $\ell(p)$ is perfect,

(ii) $[\ell(p)]_t$ is perfect,

(iii) $\ell(p) = \ell_1$

Proof: (i) is equivalent to (ii) by Theorem 3.2. We show that (i) is equivalent to (iii). It is easy to show that (iii) implies (i). Now suppose that (i) holds, that is $\ell^{\alpha\alpha}(p) = \ell(p)$ since $\ell^\alpha(p) = \ell_\infty(p)$ then $\ell^{\alpha\alpha}(p) = \ell^{\alpha\alpha}_\infty(p) = M_\infty(p)$. We shall show that $M_\infty(p) = \ell(p)$ implies inf $p_k > 0$. Suppose that $M_\infty(p) = \ell(p)$ but inf $p_k = 0$. Then there exists a strictly increasing sequence (k_i) of positive integers such that $p_{k_i} < i^{-1}$. We put

$$a_k = \begin{cases} 0 & \text{if } k \neq k_i \\ i^{-1/p_k} & \text{if } k = k_i \end{cases} \quad (i = 1,2,\ldots) \qquad (*)$$

Then for every $N>1$ we have, for $i > 2N$, $|a_k|^{p_k} = i^{-1}$ and $|a_k| N^{1/p_k} < i^{-1}$ where $k = k_i$ by (*). Therefore $a \varepsilon M_\infty(p) - \ell(p)$, contrary to the assumption that $M_\infty(p) - \ell(p)$. Hence inf $p_k > 0$, which gives us $\ell(p) = \ell_1$. It is easy to check that $M_\infty(p) = \ell_1$ if and only if inf $p_k > 0$. This completes the proof of the theorem.

Theorem 3.4 *For every* $p = (p_k)$ *we have*

(i) $c_0^\alpha(p) = M_0(p)$, where

$$M_0(p) = \bigcup_{N>1} \left\{ a = (a_k) : \sum_k |a_k| N^{-1/p_k} < \infty \right\},$$

(ii) $$[(c_0(p))_t]^\eta = \bigcup_{N>1} \left\{ a = (a_k) : \sum_k \left| \frac{a_k}{t_k} \right| N^{-1/p_k} < \infty \right\}$$

Where $\eta = \alpha$ or β,

(iii) $$\left[c_0(p)_t \right]^{\eta\eta} = \bigcap_{N>1} \left\{ x = (x_k) : \sup_k | t_k x_k | N^{1/p_k} < \infty \right\},$$

where $c_0^{\alpha\alpha}(p) = E_0$, where

$$E_0 = \bigcap_{N>1} \left\{ x = (x_k) : \sup_k |x_k| N^{1/p_k} < \infty \right\}$$

for $\eta = \alpha$ or β,

The following conditions are equivalent:

(a) $c_0(p)$ is perfect,

(b) $[c_0(p)]_t$ is perfect,

(c) $p \varepsilon c_0$.

Proof

(i) Let $a \varepsilon M_0(p)$ and $x \varepsilon c_0(p)$. Then $\Sigma_k |a_k| N^{-1/p_k} < \infty$ for some $N > 1$ and $|x_k|^{p_k} < N^{-1}$ for sufficiently large k; whence for such k it follows that

$$| a_k x_k | \leq | a_k | N^{-1/p_k}$$

so $\Sigma_k | a_k x_k | \leq \Sigma_k | a_k | N^{-1/p_k} < \infty$ and hence $M(p) \subset c_0^\alpha(p)$.
Since $c_0^\alpha(p) \subset c_0^\beta(p)$ it follows that $c_0^\alpha(p) = M_0(p)$ by Theorem (3.2) in [6,7].

(ii) Proof follows from (i), Theorem (3.1(i)) and Theorem (3.2) in [6,7].

(iii) Let $a \varepsilon E_0$ and $x \varepsilon c_0^{\alpha}(p)$. Then for every $N>1$, $|a_k|N^{1/p_k} \le K$ for all k and for some $K>0$, and $\Sigma_k |x_k| N^{1/p_k} < \infty$ for some $N>1$. Hence $|a_k x_k| \le K|x_k|N^{-1/p_k}$ which implies that $\sum_k |a_k x_k| \le K \sum_k |x_k| N^{-1/p_k} < \infty$

Consequently $a \varepsilon c_0^{\alpha\alpha}(p)$, whence $E_0 \subset c_0^{\alpha\alpha}(p)$. Since $c_0^{\alpha}(p) = c_0^{\beta}(p)$ by (i), it follows that $c_0^{\alpha\alpha}(p) \subset c_0^{\beta\beta}(p)$.

Then we have $c_0^{\alpha\alpha}(p) = E_0$, since $c_0^{\beta\beta}(p) = E_0$ by Theorem 3.2 and Theorem 3.1(ii) give us (iii). (iv) (a) is equivalent to (b) by Theorem (3.2). Since $c_0(p)$ is a β-space if and only if $p \varepsilon c_0$ and since $c_0^{\alpha\alpha}(p) = c_0^{\beta\beta}(p)$ by (c) the equivalence of (a) and (c) is immediate.

This completes the proof of the theorem.

Theorem 3.5. *For every $p = (p_k)$ we have*

(i) $\ell_0^{\alpha\alpha}(p) = M_\infty(p)$, where

$$M_\infty(p) = \bigcap_{N>1} \left\{ a = (a_k) : \sum_k |a_k| N^{1/p_k} < \infty \right\},$$

(ii) $$[\ell_\infty(p)_t]^\eta = \bigcap_{N>1} \left\{ a = (a_k) : \sum_k \left| \frac{a_k}{t_k} \right| N^{1/P_k} < \infty \right\},$$

where $[\ell_\infty(p)_t]^\eta = \bigcap_{N>1} \left\{ a = (a_k) : \sum_k |a_k| N^{1/P_k} < \infty \right\}$ for $\eta = \alpha$ or β

(iii) $\ell_\infty^{\alpha\alpha}(p) = E_\infty$, where $E_\infty = \bigcup_{N>1} \left\{ x = (x_k) : \sup_k |x_k| N^{-1/P_k} < \infty \right\}$,

(iv) $$[\langle \ell_\infty(p) \rangle_t]^{\eta\eta} = \bigcup_{N>1} \left\{ x = (x_k) : \sup_k |t_k x_k| N^{-1/P_k} < \infty \right\},$$

Where $\eta = \alpha$ or β

(v) The following conditions are equivalent:

(1) $p \varepsilon \ell_\infty$

(2) $\ell_\infty(p)$ is perfect,

(3) $[\ell_\infty(p)]_t$ is perfect

Proof (i): It is similar to the proof of Theorem 3.4(i), since $\ell_\infty^{\beta}(p) = M_\infty(p)$ Theorem 3.1(i) give us (ii).The proof of (iii) is similar to the proof of Theorem 3.4(iii), since

$\ell_{\infty}^{\beta\beta}(p) = E_{\infty}$. Theorem 3.1(ii) give us (iv). The proof of (v) is similar to the proof of Theorem 3.4(iv).

This completes the proof of the theorem.

Theorem 3.6 *For every $p = (p_k)$ we have*

(i) $c^{\alpha}(p) = c_0^{\alpha}(p) \cap \ell_1$,

(ii) $\left[\langle c(p)\rangle_t\right]^{\alpha} = \left[M_0(p)\right]_t^{*} \cap (\ell_1)_t^{*}$,

(iii) $\left[\langle c(p)\rangle_t\right]^{\beta} = \left[M_0(p)\right]_t^{*} \cap \gamma_t^{*}$,

Where $\gamma = \left\{ a = (a_k) : \sum_k a_k \text{ converges} \right\}$.

Proof (i) : Let $a \varepsilon c^{\alpha}(p) \cap \ell_1$ and $x \varepsilon c(p), |x_k - \ell|^{p_k} \to 0 (k \to \infty)$. Then $\sum_k |a_k| < \infty$ and since $x \varepsilon c(p), (x_k - \ell) \varepsilon c_0(p)$ and hence

$$\sum_k |a_k(x_k - \ell)| < \infty.$$

Now from the inequality

$$|a_k x_k| \leq |a_k(x_k - \ell)| + |\ell a_k|$$

We obtain that $\sum_k |a_k x_k| < \infty$. Therefore $a \varepsilon c^{\alpha}(p)$.

Since $c_0(p) \subset c(p)$ it follows that $c_0(p) \subset c_0^{p}(p)$. Let $a \varepsilon c^{\alpha}(p)$.

Since $e = (1,1,\ldots) \varepsilon c(p)$, it follows that $\sum_k |a_k| < \infty$, so that $a \varepsilon \ell_1$. Hence $a \varepsilon c_0^{\alpha}(p) \cap \ell_1$. This completes the proof of (i). Theorem 3.6 (i) and theorem 3.1(i) give us (ii); theorem3.1(i) give us 3.6 (iii). This completes the proof.

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