The Nepali Math. Sc. Report
Vol. 21 No. 1&2, 2003

On Complete Lifts of (1,1) Tensor Field F Satisfying Structure $F^{\upsilon+1} - \lambda^2 F^{\upsilon-1} = 0$

SUDHIR KUMAR SRIVASTAVA

Abstract: The complete lifts from a differentiable manifold M^n of class C^∞ to its cotangent bundle $T^*(M^n)$ have been studied by professor Yano and Patterson [4, 5]. Yano and Ishihara [6] studied lifts of an f-structure in the tangent and cotangent bundles. F-structures manifolds of degree $\upsilon \geq 3$ have been studied by Kim J.B. [2]. The present paper deals with some problems on complete lifts of structures mentioned above both in tangent and cotangent bundles and the prolongation in the third Tangent space $T_3(M^n)$.

1. Preliminaries

Let M^n be a differentiable manifold of class C^∞ and dimension n. Let $T^*(M^n)$ be the cotangent bundle of M^n, then $T^*(M^n)$ is also a differentiable manifold of class C^∞ and of dimension 2n. Throughout this paper we make use of the following notations and conventions.

(i) $\pi : T^*(M^n) \rightarrow (M^n)$ is the projection map of $T^*(M^n)$ onto M^n

(ii) $\Im_s^r(M^n)$ denotes the set of tensor field of class C^∞ and type (r, s) in M^n and $\Im_s^r(T^*(M^n))$ denotes the corresponding set of tensor fields in $T^*(M^n)$

(iii) Vector fields in M^n denoted by X, Y, Z . . . . . lie derivatives with respect to X is denoted by 0_X

If A is a point in M^n then $\pi^{-1}(A)$ is the fibre over A. Any point $P \in \pi^{-1}(A)$ is an ordered pair(A, P_A), where P is a 1-form in M^n and P_A is its value at A. Let U be a co-ordinate neighbourhood $\pi^{-1}(U)$ in $T^*(M^n)$, then we have

[1.1] $$(X+Y)^c = X^c + Y^c \quad \text{and}$$

[1.2] $$(F^c)(Z)^c = (F^z)^c + (0_z F)^v$$

let M^n be an n-dimensional connected differentiable manifold of class C^∞. Let there be given in M^n, a (1,1) tensor field F of class C^∞ satisfying

[1.3] $$F^{\upsilon+1} - \lambda^2 F^{\upsilon-1} = 0$$

where λ is non zero complex number. Also

rank (F) $= \frac{1}{2}$ (rank $F^{\upsilon-1}$ + dim M^n)

= r (a constant every where on M^n)

Let the operators l^* and m^* be defined as

[1.4] $$l^* \underline{\text{def}} (F/\lambda)^{\upsilon-1}, \; m^* = I - (F/\lambda)^{\upsilon-1},$$

where I denotes the identity operator on M^n. Then the operators I* and m* applied to the tangent space at a point of the manifold be complementary projection operators.

Agreement [1.1]

In what follows we make use of the following results [6] for any $X, Y \in \Im_0^1(M^n)$ we have

[1.5] (a) $[X^c, Y^c] = [X,Y]^c$ and

(b) $F^c X^c = [FX]^c$

F-Structure manifold of degree υ (≥ 3)

Let F be a non zero tensor field of type (1,1) and of class C^∞ on an n-dimensional M^n such that [2]

[1.6] $$F^\upsilon + (-1)^{\upsilon-1} F = 0 \quad \text{and}$$

$$F^u + (-1)^{u-1} F \neq 0 \quad \text{for } 1 < u < \upsilon$$

Where υ is fixed positive integer greater than 2. Such a structure on M^n is called an F-structure of rank r and degree υ. If the rank of F is constant and equal to r, then M^n is called F-structure manifold of degree υ (≥ 3). The case when υ is odd has been considered here.

Let the operators on M^n be defined as follows [2]

[1.7] $$l = -(-1)^{\upsilon+1}\frac{F^{\upsilon+1}}{\lambda^2} \quad \text{and}$$

$$m = I + (-1)^{\upsilon+1}\frac{F^{\upsilon+1}}{\lambda^2},$$

where I denotes the identity operator on M^n

From the operators defined by [1.7] we have

[1.8] $$l + m = I,\ l^2 = l \text{ and } m^2 = m$$

For F satisfying [1.6] there exists complementary distribution L and M corresponding to the projection operators l and m respectively

If rank (F) be r, constant on M^n then

dim L = r and

dim M = n − r

We have the following results

[1.9] (i) $Fl = lF = F$ and

$Fm = mF = 0$

(ii) $F^{\upsilon-1} = -\lambda^2 l$ and

$F^{\upsilon-1}m = 0$

2. The Complete lift of F in the tangent Bundle T(M^n)

The complete lifts F^c of an element of $\Im^1_1(M^n)$ with local component F^h_i has components of the form [6]

[21] $$F^c = \begin{pmatrix} F^h_i & 0 \\ 2F^h_i & F^i_h \end{pmatrix}$$

Now we prove some theorems on the complete lifts of $F((\upsilon+1),\ \lambda^2(\upsilon-1))$– Structure satisfying [1.3].

Theorem 2.1. *The complete lift of a* $F((\upsilon+1),\ \lambda^2(\upsilon-1))$- *Structure also has* $F((\upsilon+1),\ \lambda^2(\upsilon-1))$- *Structure in the tangent bundle.*

Proof : Let F, G ∈ $\Im^1_1(M^n)$ then we have

[2.2] $$(FG)^c = F^cG^c$$

Putting F = G we obtain

[2.2] $$(F^2)^c = (F^c)^2$$

Putting $G = F^2$ in [2.2] and making use of [2.3] we get

$$(F^3)^c = (F^c)^3$$

Continuing the above process of replacing G in equation [2.2] by some higher degree of F we obtain

[2.4] $$(F^{\upsilon})^c = (F^c)^{\upsilon}$$

where υ is any positive integer. Taking complete lift on both side of equation [1.3] we get

$$(F^{\upsilon+1})^c - (\lambda^2 F^{\upsilon-1})^c = 0$$

which in view of the equation [2.4] gives

$$(F^c)^{\upsilon+1} - \lambda^2 (F^c)^{\upsilon-1} = 0$$

Thus the complete lift of F also has $F((\upsilon+1), \lambda^2(\upsilon-1))^{\upsilon-1}$-Structure in $T(M^n)$

The complete lift l^{*c} and m^{*c} of l^* and m^* are complementary projection tensor in $T(M^n)$. Thus there exist in $T(M^n)$ two complementary distributions L^{*c} and M^{*c} determined by l^{*c} and m^{*c} respectively.

Theorem 2.2. *The complete lift* M^{*c} *of the distribution* M^* *in* $T(M^n)$ *is integrable, iff* M^* *is integrable in* M^n.

Proof: It is well known that the distribution M^* is integrable in M^n iff

[2.6] $$l^*[m^*X, m^*Y] = 0$$

taking complete lifts on both sides we get

[2.7] $$l^{*c}[m^{*c}X^c, m^{*c}Y^c] = 0$$

Where

$$l^{*c} = (I - m)^{*c} = I - m^{*c} \text{ as } I^c = I$$

In consequence of equation [2.7] M^{*c} is integrable in $T(M^n)$

In the same way we can proof the theorem

Theorem 2.3. *The complete lift* L^{*c} *of* l^* *in* $T^*(M^n)$ *is integrable iff* L^* *integrable in* M^n.

Theorem 2.4. *The structure* F^c *is partially integrable iff* F *is partially integrable in* M^n.

Proof: We know that F is partially integrable iff

[2.8] $$N(l^*X, l^*Y) = 0$$

Taking complete lifts on both sides we obtain

[2.9] $$N^c(l^{*c} X^c, l^{*c} Y^c) = 0$$

Hence F^c is partially integrable iff F is partially integralbe in M^n.

Theorem 2.5. *For any* X, Y $\in \Im_0^1(M^n)$, *let* F *be integrable in* M^n, *Thus* F^c *is integrable in* $T^*(M^n)$ *iff*

$$N^c(X^c, Y^c] = 0.$$

Proof : We know that F is integrable iff

[2.10] $$N(X,Y) = 0$$

where N(X,Y), the Nijenhuis tensor of F satisfying [1.3] and it is given by [6]

[2.11] $$N_{F,F}(X,Y) = [FX, FY] - F[FX, Y] - F[X, FY] + F^2[X,Y]$$

Taking complete lift of both sides we have

[2.12] $$N^c(X^c, Y^c) = [F^cX^c, F^cY^c] - F^c[F^cX^c, Y^c] - F^c[X^c, F^cY^c] + (F^2)^c[X^c, Y^c]$$

also taking complete lift of [2.10] we get

$$N^c(X^c, Y^c] = 0.$$

Which is view of Equation [2.11] and [2.12] and the fact F is integrable in M^n shows that F^c is integrable in $T(M^n)$

3. The compete lift of F((υ+1), $\lambda^2(\upsilon-1)$-Structure in Cotangent Bundle

In this section we prove some theorems on complete lift of F satisfying F((υ+1), $\lambda^2(\upsilon-1)$-Structure.

Theorem 3.1. *The Nijenhuis tensor of the complete lift of* $F^{\upsilon+1}$ *vanishes if the lie derivative of the tensor field* $F^{\upsilon+1}$ *with respect to* X *and* Y *are both zero and* F *is an almost* π *- structure on* M^n.

Proof : In consequence of (2-11) the Nijenhuis tensor of $F^{\upsilon+1}$ is given by

$$[3.1] \quad N_{(F^{\upsilon+1})^c (F^{\upsilon+1})^c}(X^c, Y^c) = [(F^{\upsilon+1})^c X^c, (F^{\upsilon+1})^c X^c] - (F^{\upsilon+1})^c [(F^{\upsilon+1})^c X^c, Y^c] - (F^{\upsilon+1})^c [X^c, (F^{\upsilon+1})^c Y^c] + (F^{\upsilon+1})^c (F^{\upsilon+1})^c [X^c, Y^c]$$

which in view of [1.3] takes the form

$$[3.2] \quad N_{(F^{\upsilon+1})^c (F^{\upsilon+1})^c}(X^c, Y^c) = \lambda^4 [(F^{\upsilon-1})^c X^c, (F^{\upsilon-1})^c Y^c] - \lambda^4 (F^{\upsilon-1})^c [(F^{\upsilon-1})^c X^c, Y^c] - \lambda^4 (F^{\upsilon-1})^c [X^c, (F^{\upsilon-1})^c Y^c] + \lambda^4 (F^{\upsilon-1})^c (F^{\upsilon-1})^c [X^c, Y^c].$$

In Consequence of [1.2] we have

$$[3.3] \quad (F^{\upsilon-1})^c X^c = (F^{\upsilon-1} X)^c X^c + (\mathcal{L}_x F^{\upsilon-1})^v$$

Hence we get

$$[3.4] \quad N_{(F^{\upsilon+1})^c (F^{\upsilon+1})^c}(X^c, Y^c) = [(F^{\upsilon-1} X)^c, (F^{\upsilon-1} Y)^c] + [(\mathcal{L}_x F^{\upsilon-1})^v (F^{\upsilon-1}, Y^c] + [(F^{\upsilon-1} X)^c, (\mathcal{L}_y F^{\upsilon-1})^v] + [(\mathcal{L}_y F^{\upsilon-1})^v, (\mathcal{L}_y F^{\upsilon-1})^v] - [(F^{\upsilon-1})^c, [(F^{\upsilon-1} X)^c, Y^c] - (F^{\upsilon-1})^c, [(\mathcal{L}_y F^{\upsilon-1} X)^v, Y^c] - (F^{\upsilon-1})^c, [X^c, (F^{\upsilon-1}, Y)^c] - (F^{\upsilon-1})^c, [X^c, (\mathcal{L}_y F^{\upsilon-1})^v] + (F^{\upsilon-1})^c, (F^{\upsilon-1})^c, [X^c, Y^c]$$

If the lie derivatives of the tensor field $F^{\upsilon-1}$ with respect to X and Y are both zero we have

$$\mathcal{L}_x F^{\upsilon-1} = 0 \quad \text{and} \quad \mathcal{L}_y F^{\upsilon-1} = 0$$

Therefore equation [3.4] takes the form

[3.5] $$N_{(F^{\upsilon+1})^c(F^{\upsilon+1})^c}(X^c, Y^c) = [(F^{\upsilon-1}X)^c, (F^{\upsilon-1}Y)^c] - (F^{\upsilon-1})^c[(F^{\upsilon-1}X)^c, Y^c] - (F^{\upsilon-1})^c[X^c, (F^{\upsilon-1}Y)^c] + (F^{\upsilon-1})^c(F^{\upsilon-1})^c[X^c, Y^c]$$

In view of equation [1.5] the equation [3.5] reduces to

[3.6] $$N_{(F^{\upsilon+1})^c(F^{\upsilon+1})^c}(X^c, Y^c) = +[F^{\upsilon-1}X, F^{\upsilon-1}Y]^c - (F^{\upsilon-1})^c[F^{\upsilon-1}X, Y]^c - (F^{\upsilon-1})^c[X, F^{\upsilon-1}Y]^c + (F^{\upsilon-1})^c(F^{\upsilon-1})^c[X, Y]^c$$

Let F be an almost π-structure on M^n then $F^2 = \lambda^2 I$, where I is unit tensor field. Hence $F^{\upsilon-1} = \lambda^2$ I or I and therefore [3.6] takes the form

$$N_{(F^{\upsilon+1})^c(F^{\upsilon+1})^c}(X^c, Y^c) = [X, Y]^c - [X, Y]^c - [X, Y]^c + [X, Y]^c = 0$$

Theorem 3.2. *The Nijenhuis tensor of the complete lift of* $F^{\upsilon+1}$ *is equal to* λ^4 *multiplied by the complete lift of Nijenhuis tensor of* $F^{\upsilon+1}$, *if*

(3.7) (i) $\mathcal{L}_X F^{\upsilon-1} = 0$, $\mathcal{L}_Y F^{\upsilon-1} = 0$ and

(ii) $[X, Y]^c = 0$, $\tilde{\upsilon} = 0$

Where

$$\tilde{\upsilon} \ \underline{\underline{\text{dif}}} \ \mathcal{L}_{[F^{\upsilon-1}X, Y]}F^{\upsilon-1} + \mathcal{L}_{[X, F^{\upsilon-1}Y]}F^{\upsilon-1} - \mathcal{L}_{[X,Y]}F^{2\upsilon-2}$$

Proof : In view of equation [1.1] and [2.11] we have

[3.8] $$N_{F^{\upsilon-1}F^{\upsilon-1}}(X, Y)^c = [F^{\upsilon-1}X, F^{\upsilon-1}Y]^c - (F^{\upsilon-1})[F^{\upsilon-1}X, Y]^c - (F^{\upsilon-1}[X, F^{\upsilon-1}Y])^c + (F^{\upsilon-1}F^{\upsilon-1}[X, Y])^c$$

Which on account of [3.3] yield

$$N_{F^{\upsilon-1}F^{\upsilon-1}}(X, Y)^c = [F^{\upsilon-1}X, F^{\upsilon-1}Y]^c - (F^{\upsilon-1})^c[F^{\upsilon-1}X, Y]^c - (\mathcal{L}_{[F^{\upsilon-1}X, Y]}F^{\upsilon-1})^v - (F^{\upsilon-1})^c[X, F^{\upsilon-1}Y]^c - (\mathcal{L}_{[X, F^{\upsilon-1}Y]}F^{\upsilon-1})^v + (F^{2\upsilon-2})^c[X, Y]^c - (\mathcal{L}_{[X,Y]}F^{2\upsilon-2})^v$$

But we have [6]

[3.9] $$(\mathrm{F}^{\upsilon-1})^c\,(\mathrm{F}^{\upsilon-1})^c = (\mathrm{F}^{2\upsilon-2})^c + [\mathrm{N}_{\mathrm{F}^{\upsilon-1}\,\mathrm{F}^{\upsilon-1}}]^v$$

Hence in view of [3.9], the equation [3.8] becomes

[3.10] $$\mathrm{N}_{\mathrm{F}^{\upsilon-1}\mathrm{F}^{\upsilon-1}}(X,Y)^c = [\mathrm{F}^{\upsilon-1}X, \mathrm{F}^{\upsilon-1}Y]^c - (\mathrm{F}^{\upsilon-1})^c\,[\mathrm{F}^{\upsilon-1}X, Y]^c$$
$$- (\mathrm{F}^{\upsilon-1})^c\,[X, \mathrm{F}^{\upsilon-1}Y]^c + (\mathrm{F}^{2\upsilon-2})^c\,[X,Y])^c$$
$$- (\mathcal{L}_{[\mathrm{F}^{\upsilon-1}X,Y]}\,\mathrm{F}^{\upsilon-1})^v + \mathcal{L}_{[X,\mathrm{F}^{\upsilon-1}Y]}\,\mathrm{F}^{\upsilon-1})^v$$
$$- (\mathcal{L}_{[X,Y]}\,\mathrm{F}^{2\upsilon-2})^v$$

Now from [3.9] we have

$$(\mathrm{F}^{2\upsilon-2})^c = (\mathrm{F}^{\upsilon-1})^c\,(\mathrm{F}^{\upsilon-1})^c - (\mathrm{N}_{[\mathrm{F}^{\upsilon-1},\mathrm{F}^{\upsilon-1}]})^v$$

Thus

[3.11] $$\mathrm{N}_{\mathrm{F}^{\upsilon-1}\mathrm{F}^{\upsilon-1}}(X,Y)^c = [(\mathrm{F}^{\upsilon-1}X, \mathrm{F}^{\upsilon-1}Y]^c - (\mathrm{F}^{\upsilon-1})^c\,[\mathrm{F}^{\upsilon-1}X, Y]^c$$
$$- (\mathrm{F}^{\upsilon-1})^c\,[X, \mathrm{F}^{\upsilon-1}Y]^c + (\mathrm{F}^{\upsilon-1})^c\,(\mathrm{F}^{\upsilon-1})^c\,[X,Y]^c$$
$$- (\mathrm{N}_{[\mathrm{F}^{\upsilon-1},\mathrm{F}^{\upsilon-1}]})^v\,(X,Y^c) + (\mathcal{L}_{[\mathrm{F}^{\upsilon-1}X,Y]}\,\mathrm{F}^{\upsilon-1})^v$$
$$+ (\mathcal{L}_{[X,\mathrm{F}^{\upsilon-1}Y]}\,\mathrm{F}^{\upsilon-1})^v - (\mathcal{L}_{(X,Y)}\,\mathrm{F}^{2\upsilon-2})^v$$

In view of equation [3.11] the equation [3.5] takes the form

$$\mathrm{N}_{(\mathrm{F}^{\upsilon+1})^c(\mathrm{F}^{\upsilon+1})^c}(X^c,Y^c) = \mathrm{N}_{\mathrm{F}^{\upsilon-1}\mathrm{F}^{\upsilon-1}}(X,Y)^c + (\mathrm{N}_{[\mathrm{F}^{\upsilon-1},\mathrm{F}^{\upsilon-1}]})^v\,[X,Y]^c$$
$$- \{\mathcal{L}_{[\mathrm{F}^{\upsilon-1}X,Y]}\,\mathrm{F}^{\upsilon-1} - \mathcal{L}_{[X,\mathrm{F}^{\upsilon-1}Y]}\,\mathrm{F}^{\upsilon-1}$$
$$+ \mathcal{L}_{(X,Y)}\,\mathrm{F}^{2\upsilon-2}\}^v$$

In consequence of [3.7] we have

[3.12] $$(\mathrm{N}_{(\mathrm{F}^{\upsilon+1})^c(\mathrm{F}^{\upsilon+1})^c}(X^c,Y^c) = \lambda^4\,(\mathrm{N}_{[\mathrm{F}^{\upsilon-1},\mathrm{F}^{\upsilon-1}]}(X,Y)^c) + \lambda^4(\mathrm{N}_{[\mathrm{F}^{\upsilon-1},\mathrm{F}^{\upsilon-1}]})^v$$
$$[X,Y]^c - \tilde{\upsilon}^v$$

Let $[X,Y]^c = 0$ and $\tilde{\upsilon}^v = 0$ then [3.12] reduces to

$$(\mathrm{N}_{(\mathrm{F}^{\upsilon+1})^c(\mathrm{F}^{\upsilon+1})^c}(X^c,Y^c) = \lambda^4\,(\mathrm{N}_{[\mathrm{F}^{\upsilon-1},\mathrm{F}^{\upsilon-1}]}(X,Y)^c)$$

Theorem 3.3. *The Nijenhuis tensor of the complete lift of* $\mathrm{F}^{\upsilon-1}$ *is equal to the complete lift of the Nijenhuis tensor of* $\mathrm{F}^{\upsilon-1}$ *if*

(i) $\mathcal{L}_X F^{\upsilon-1} = 0, \quad \mathcal{L}_Y F^{\upsilon-1} = 0$

and

(ii) $\mathcal{L}_X Y = 0 \qquad \tilde{\upsilon}^{v} = 0$

Proof : Since $[X, Y]^c = 0$ implies that

$$[X, Y] = 0 \quad \text{if} \quad \mathcal{L}_X \ Y = 0,$$

Therefore from [3.2] the result follows.

Theorem 3.4. *The process of computing the Nijenhuis tensor of* $F^{\upsilon-1}$ *and taking complete lift are commutative.*

Proof: Theorem follows easily with the help of [3.1] and above theorem.

4. Prolongation of a $F((\upsilon+1), -\lambda^2(\upsilon-1))$-Structure in third tangent space $T_3(M^n)$

Let us denote $T_3(M^n)$ the third order tangent bundle over M^n and let F^{III} be the third lift on F in $T_3(M^n)$ then we have

For any F, $G \in \Im^1_1 M^n$ the following holds

[4.1] $$(G^{III} F^{III}) X^{III} = G^{III}(F^{III} X^{III})$$
$$G^{III}(FX)^{III}$$
$$(G(FX))^{III}$$
$$(GF)^{III} X^{III}$$

For every $X \in \Im^1_0 (M^n)$, therefore we have

$$G^{III} F^{III} = (GF)^{III}$$

If P(t) denotes a polynomial of variable then we have

[4.2] $$(P(F))^{III} = P(F)^{III}$$

where $$F \in \Im^1_1 (M^n)$$

Theorem 4.1. *The third lift* F^{III} *defines* a $F((\upsilon+1), -\lambda^2(\upsilon-1))$*-structure in* $T_3(M^n)$ *iff* F *defines a* $F((\upsilon+1), -\lambda^2(\upsilon-1))$*-structure in* M^n.

Proof : Let F satisfy [1.3] then F defines a $F((\upsilon+1), -\lambda^2(\upsilon-1))$- structure in M^n satisfying $F^{\upsilon+1} - \lambda^2 F^{\upsilon-1} = 0$.
which in view of [4.2] takes the form

[4.3] $$(F^{III})^{\upsilon+1} - \lambda^2 (F^{III})^{\upsilon-1} = 0$$

Therefore F^{III} defines $F((\upsilon+1), -\lambda^2(\upsilon-1))$ structure in $T_3(M^n)$

Theorem 4.2. *The third lift* F^{III} *is integrable in* $T_3(M^n)$ *iff F is integrable in* M^n

Proof: Let us denote N^{III} and N Nijenhuis tensors of F^{III} and F, respectively. Then we have [6]

[4.4] $$N^{III}(X,Y) = (N(X,Y))^{III}$$

We know that $F((\upsilon+1), -\lambda^2(\upsilon-1))$ structure is integrable in M^n iff $N(X,Y) = 0$

Which in view of [4.4] is equivalent to

[4.5] $$N^{III}(X,Y) = 0$$

Thus F^{III} is integrable iff F is integrable in M^n

Theorem 4.3. *The third lift* F^{III} *of F is partially integrable in* $T_3(M^n)$ *iff F is partially integrable in* M^n.

Proof: We know that For F to be partially integrable in M^n the following holds

$N(l^* X, l^* Y) = 0$ and $N(m^* X, m^* Y) = 0$

Which in view of Equation [4.4] takes the form

[4.6] $$N^{III}(l^{*III} X^{III}, l^{*III} Y^{III}) = 0 \quad \text{and}$$
$$N^{III}(m^{*III} X^{III}, m^{*III} Y^{III}) = 0$$

Where l^{*III}, m^{*III} are operators in $T_3(M^n)$ which defines the distributions L^{*III} and M^{*III} respectively. Thus the equation [4.6] gives the condition for F^{III} to be partially integrable.

REFERENCES

[1] Ishihara, S. and Yano K., *On integrability condition of a structure satisfying* $f^3 + f = 0$, Quart. Jour. Math Oxford Vol 25, 217–222.

[2] Kim. J. B., *Notes on f-manifold*, Tensor N-S Vol. 29, 299–302.

[3] Nivas Ram and Srivastava, S. K., *On horizontal and complete lifts from a manifold with* $f_\lambda(7,-1)$ *structure to its contagent bundle* Jour. Tensor Soc-India, Vol . 14, 42–48.

[4] Yano, K., and Patterson, E. M., *Vertical and complete lift from a manifold to its contagent bundle*. Jour. Math. Soc. Japan Vol. 19. No. 1, 91–113.

[5] Ram Nivaş, Sahadat Ali, Sudhir Kumar Srivastava, *On horizontal & complete lift of* (1,1) *tensor fields F satisfying an* $F^n + a_{n-1} F^{n-1} + \ldots + a_2F^2 + a_1F = 0$ *structure*, *Demonstratio Mathematica* Vol. XXXV No. 1,155–163.

[6] Yano, K., and Ishihara, S., *Tangent and Contangent bundles*, Marcel Dekker Inc. New York

SUDHIR KUMAR SRIVASTAVA
Govt. P.G. College
Dhamtari (C.G.) INDIA
Pin_Code - 493 773.