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On the Cauchy Problem For a Sobolev Type System in Hydrodynamics of Rotating Fluid with Heat Transfer

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Abstract: The solution and properties of various equations or systems of equations in mathematical physics are generally applied in different fields of natural sciences, such as in the science of oceans, prognosis of weather, theory of hydro-nuclear reactors, etc. One of such systems of equations is the Sobolev system studied by eminent Russian mathematician S.L. Sobolev. Here the solution of Sobolev type homogeneous system of partial differential equations with initial conditions, where the heat transfer is also taken into account, has been constructed in explicit form. For the construction, basically, Fourier-Laplace transformations have been applied. Then, using Duhamel's principle, the solution of the corresponding non-homogeneous system has been found. In the process of investigation, the uniqueness of the obtained solution has been proved and the estimation of the solution in Sobolev space is established by using Mareinkiewicz theorem on multiplicators. The central part is the study of asymptotic behavior of the solution for large time. Remarkable results have been obtained by investigating the improper multiple integral depending on parameters. For this, mainly, the method of stationary phase is used.

1. Introduction

We know that the construction and investigation of mathematical models of physical phenomena constitute the subject of mathematical physics. The solutions and properties of various equations or systems of equations in mathematical physics are generally applied in different fields of natural sciences, such as in the science of Oceans, prognosis of weather, theory of hydro-nuclear reactors, etc. Our everyday life is full of examples of fluid motion, for instance, stirring a cup of tea, flows in rivers, ocean waves, hurricanes and so on. The equations that describe the most fundamental behavior of an inviseid fluid were derived by Euler two and half centuries ago in 1755. Incorporation of the effects of viscosity leads to versions of Euler equations, called Navier-Stokes equations.The idea of wide application of mathematical models of rotating fluid to the study of atmospheric processes belongs to Russian mathematician A. A. Friedman. In the beginning of 20th century. Friedman contributed a series of fundamental works in dynamics of atmospheric processes. Later on, different types of Cauchy problems and initial boundary value problems for hydrodynamics system were studied by various mathematicians, especially, S. L. Sobolev, V. P. Mikhailov, O. A. Ladyzhenskaya, V.N. Maslennikova, M.E. Bogovskii. Eminent Russian mathematicians S. L. Sobolev initiated the study of a system of partial differential equations during the World War II, when it became necessary to study the stability of trajectory for rotating projectile filled with fluid. Now a days, this system is known as Sobolev system and has the following form [10].

$$(1)\qquad \begin{cases} \dfrac{\partial \vec{v}}{\partial t} - [\vec{v}, \vec{\omega}] + \nabla P = \vec{F}(x,t), & x \in \Omega\ \mathbb{R}^2,\ t \geq 0 \\ \operatorname{div} \vec{v} = 0: \end{cases}$$

where $\vec{v}$ – velocity fields

$\vec{\omega}$ – angular velocity of rotation fluid

p – pressure

t – time

$\vec{F}$ – mass density of external forces

V. N. Maslennikova, one of the former research students of S. L. Sobolev, studied the asymptotic behavior of solutions of different linearized systems of hydrodynamics of rotating fluids with and without the consideration of compressibility and viscosity [6, 7, 8, 9]. M. E. Bogovskii, a student of Maslennikova, also studied and is still continuing the study of various types of boundary value problems in hydrodynamics.The study of a Sobolev type system with heat transfer has even more practical applications than the Sovolev system itself, M. L. Marchhuk introduced such a system [5] in his book "Mathematical Models of Circulation in Oceans" in 1980, in which a numerical approach is suggested for solution. Here, a Sobolev type system in hydrodynamics with account of heat transfer is taken under consideration. The solution of a Cauchy problem for the system is constructed in explicit form. In the process of investigating the solution, uniqueness theorem is established for the Cauchy problem solution and the solution is estimated in Sobolev spaces. The most important part of the work is the study of asymptotic behavior of the solution for large time. The system undertaken for study is the following :

$$(2)\qquad \begin{cases} \dfrac{\partial v_1}{\partial t} - \omega v_2 + \dfrac{\partial P}{\partial x_1} = f_1 \\ \dfrac{\partial v_2}{\partial t} - \omega v_1 + \dfrac{\partial P}{\partial x_2} = f_2 \\ \dfrac{\partial v_3}{\partial t} - \sigma T + \dfrac{\partial P}{\partial x_3} = f_3 \\ \dfrac{\partial T}{\partial t} - \gamma v_3 = f \\ \dfrac{\partial v_1}{\partial x_1} - \dfrac{\partial v_2}{\partial x_2} + \dfrac{\partial v_3}{\partial x_3} = 0 \end{cases}$$

where

v_1, v_2, v_3 – components of velocity $\vec{v}$

p –pressure

T–temperature (deviation of temperature from some standard value T_0 corresponding to the plane $x_3 = 0$)

σ–free convection coefficient (positive constant)

γ–mean gradient of density (positive constant)

ω–constant vector of angular velocity

f_1, f_2, f_3 – components of mass density $\vec{F}$ of external forces

f– heat source density

Without any loss of generality, we can take $\vec{\omega} = (0, 0, \omega)$, The homogeneous system corresponding to (2) is as follows:

$$(2') \qquad \begin{cases} \dfrac{\partial v_1}{\partial t} - \omega v_2 + \dfrac{\partial P}{\partial x_1} = 0 \\ \dfrac{\partial v_2}{\partial t} + \omega v_1 + \dfrac{\partial P}{\partial x_2} - 0 \\ \dfrac{\partial v_3}{\partial t} + \sigma T + \dfrac{\partial P}{\partial x_3} = 0 \\ \dfrac{\partial T}{\partial t} - \gamma v_3 = 0 \\ \dfrac{\partial v_1}{\partial x_1} + \dfrac{\partial v_2}{\partial x_2} + \dfrac{\partial v_3}{\partial x_3} = 0 \end{cases}$$

The solution of (2) is considered in the domain $\Omega = \{(x, t) : x \in \mathbb{R}^3,\ t \geq 0\}$ with the following initial conditions :

$$(3) \qquad \begin{cases} \vec{v}(x,t)\big|_{t=0} = \vec{v}^0(x) \\ T(x,t)\big|_{t=0} = T^0(x) \\ \operatorname{div}\ \vec{v}^0 = (x) \end{cases}$$

The main results of the work are given in the form of four theorems. Theorem 1 is on the explicit solution and other theorems are on the properties of the solution.

2. Construction of Solution

The solution of the Cauchy problem (2′), (3) for homogeneous system is first constructed in explicit form [1]. For this, basically, Fourier-Laplace transformations have been applied. Then, using Duhamel's principle, the solution of the corresponding non-homogeneous system is found.

In the process of construction of solution different special cases are considered. For the most general case, the following kernels are found.

$$(4)\qquad \begin{cases} \mathcal{K}_1(x,t) = \frac{1}{2\pi^2 r} \int\limits_0^{\pi/2} \cos(t\, g(\psi))d\psi \\ \mathcal{K}_2(x,t) = \frac{1}{2\pi^2 r} \int\limits_0^{\pi/2} \frac{\sin(t\, g(\psi))}{g(\psi)} d\psi \\ \mathcal{K}_3(x,t) = \frac{1}{2\pi^2 r} \int\limits_0^{\pi/2} \frac{[1-\cos(t\, g(\psi))]}{[g(\psi)]^2} d\psi. \end{cases}$$

where

$$g(\psi) = \sqrt{(\omega^2 - \sigma\gamma)\left(\tfrac{\rho}{r}\right)^2 \sin^2\psi + \sigma\gamma}\,,$$
$$\rho = |x'|\,.\; x' = (x_1, x_2)\,.\; r = |x|\,.\; x = (x_1, x_2, x_3)\,.$$

By the help of these kernels, the solution of (2′), (3) is written. It is given by

$$\begin{aligned}(5a)\quad v_1(x,t) = &\int\limits_{\mathbf{R}^3} -\Delta v_1^0(y)\mathcal{K}_1(x-y,t)dy \\ &+ \int\limits_{\mathbf{R}^3}\left\{\omega\frac{\partial^2 v_2^0(y)}{\partial y_3^2} - \omega\frac{\partial^2 v_2^0(y)}{\partial y_2 \partial y_3} + \sigma\frac{\partial^2 T^0(y)}{\partial y_1 \partial y_3}\right\}\times\mathcal{K}_2\,(x-y,t)\,dy \\ &+ \int\limits_{\mathbf{R}^3}\left\{\sigma\gamma\frac{\partial^2 v_1^0(y)}{\partial y_3^2} - \sigma\gamma\frac{\partial^2 v_2^0(y)}{\partial y_1 \partial y_2} + \sigma\omega\frac{\partial^2 T^0(y)}{\partial y_2 \partial y_3}\right\}\times\mathcal{K}_3\,(x-y,t)\,dy\end{aligned}$$

$$\begin{aligned}(5b)\quad v_2(x,t) = &\int\limits_{\mathbf{R}^3} -\Delta v_2^0(y)\mathcal{K}_1(x-y,t)dy \\ &+ \int\limits_{\mathbf{R}^3}\left\{-\omega\frac{\partial^2 v_1^0(y)}{\partial y_3^2} - \omega\frac{\partial^2 v_3^0(y)}{\partial y_1 \partial y_3} + \sigma\frac{\partial^2 T^0(y)}{\partial y_2 \partial y_3}\right\}\times\mathcal{K}_2\,(x-y,t)\,dy \\ &+ \int\limits_{\mathbf{R}^3}\left\{\sigma\gamma\frac{\partial^2 v_1^0(y)}{\partial y_1 \partial y_2} + \sigma\gamma\frac{\partial^2 v_2^0(y)}{\partial y_1^2} - \sigma\omega\frac{\partial^2 T^0(y)}{\partial y_1 \partial y_3}\right\}\times\mathcal{K}_3\,(x-y,t)\,dy\end{aligned}$$

$$\begin{aligned}(5c)\quad v_3(x,t) = &\int\limits_{\mathbf{R}^3} -\Delta v_3^0(y)\mathcal{K}_1(x-y,t)dy \\ &+ \int\limits_{\mathbf{R}^3}\left\{\omega\frac{\partial^2 v_1^0(y)}{\partial y_2 \partial y_3} - \omega\frac{\partial^2 v_2^0(y)}{\partial y_1 \partial y_3} - \sigma\left(\frac{\partial^2}{\partial y_1^2} + \frac{\partial^2}{\partial y_2^2}\right)T^0(y)\right\}\mathcal{K}_2(x-y,t)dy\end{aligned}$$

$$(5d)\quad P(x,T) = \int\limits_{\mathbf{R}^3}\left\{\omega\frac{\partial^2 v_1^0(y)}{\partial y_2} - \omega\frac{\partial v_2^0(y)}{\partial y_1} + \sigma\frac{\partial T^0(y)}{\partial y_3}\right\}\mathcal{K}_1\,(x-y,t)\,dy$$

$$-\int_{\mathbf{R}^3}\left\{\sigma\gamma\frac{\partial v_1^0(y)}{\partial y_1}+\sigma\gamma\frac{v_2^0(y)}{\partial y_2}+\omega^2\frac{\partial v_3^0(y)}{\partial y_3}\right\}\mathcal{K}_2(x-y,t)\,dy$$

$$+\,\omega\int_{\mathbf{R}^3}\left\{\sigma\gamma\frac{v_1^0(y)}{\partial y_2}-\sigma\gamma\frac{\partial v_2^0(y)}{\partial y_1}+\sigma\omega\frac{\partial T^0(y)}{\partial y_3}\right\}\mathcal{K}_3(x-y,t)\,dy$$

(5e) $$T(x,t)=\int_{\mathbf{R}^3}-\Delta T^0(y)\mathcal{K}_1(x-y,t)dy-\gamma\int_{\mathbf{R}^3}-\Delta v_3^0(y)\mathcal{K}_2(x-y,t)dy$$

$$+\,\omega\int_{\mathbf{R}^3}\left\{\gamma\frac{\partial^2 v_1^0(y)}{\partial y_2\partial y_3}-\gamma\frac{\partial^2 v_2^0(y)}{\partial y_1\partial y_3}+\omega\frac{\partial^2 T^0(y)}{\partial v_3^2}\right\}\mathcal{K}_3(x-y,t)\,dy.$$

Now, the solution of the non-homogeneous system (2) with the same initial conditions, i.e., the solution of the Cauchy problem (2), (3) is found from the solutions (5) of the corresponding homogeneous system by using the Duhamel's principle.

In this connection, for the external force $\vec{F}=(f_1,f_2,f_3,)\in L_2(R^3)$, we assume, without loss of generality that

$$\operatorname{div}\vec{F}=0$$

The solution of the Cauchy problem (2), (3) has the following form:

$$v_1^*(x,t)=v_1(x,t)+\widetilde{v}_1(x,t)$$

where

(6) $$\widetilde{v}_1(x,t)=\int_0^t\int_{\mathbf{R}^3}-\Delta f_1(y,\tau)\mathcal{K}_1(x-y,t-\tau)\,dy\,d\tau$$

$$+\int_0^t\int_{\mathbf{R}^3}\left\{\omega\frac{\partial^2 f_2(y,\tau)}{\partial y_3^2}-\omega\frac{\partial^2 f_3(y,\tau)}{\partial y_2\partial y_3}+\sigma\frac{\partial^2 f(y,\tau)}{\partial y_1\partial y_3}\right\}\mathcal{K}_2(x-y,t-\tau)\,dy\,d\tau$$

$$+\int_0^t\int_{\mathbf{R}^3}\left\{\sigma\gamma\frac{\partial^2 f_1(y,\tau)}{\partial y_2^2}-\sigma\gamma\frac{\partial^2 f_2(y,\tau)}{\partial y_1\partial y_2}+\sigma\omega\frac{\partial^2 f(y,\tau)}{\partial y_2\partial y_3}\right\}\mathcal{K}_3(x-y,t-\tau)\,dy\,d\tau$$

Expressions for $v_2^*(x,t)$, $v_3^*(x,t)$, $P^*(x,t)$ and $T^*(x,t)$ are found in the same way as that in finding $v_1^*(x,t)$.

So, we have the following result.

Theorem 1. *Let the initial data $\vec{v}^0(x)$ and $T^0(x)$ in* (3) *be sufficiently smooth and decrease as $|x|\to\infty$. Then the solution of* (2'), (3) *for $\omega^2\ge\sigma\gamma$ is given by* (5a) – (5c). *The solution of* (2). (3) *with additional condition div $\vec{F}=0$, where $\vec{F}=(f_1,f_2,f_3)$ is as follows*

$$v_1^*(x,t) = v_1(x,t) + \tilde{v}_1(x,t),$$

where $\tilde{v}_1(x,t)$ is given by (6). Other components $v_2^*(x,t)$, $v_3^*(x,t)$, $P^*(x,t)$ have similar forms

2. Uniqueness and Estimation of Solution

In the process of investigation of the obtained solution, uniqueness theorem is proved and then the estimates of solution are established in Sobolev spaces by using Marcinkiewez theorem on multiplicators [4].

For Cauchy problem (2′), (3), the following uniqueness theorem holds [2].

Theorem 2 *The solutions $\vec{v}(x,t)$ and $T(x,t)$ of the Cauchy problem* (2′), (3) *are given in L_2, while the solution $P(x,t)$ is determined up to a function of t. In addition, ∇P is again unique in L_2.*

The following theorem on the estimation of the solutions holds true [2].

Theorem 3. *If the initial data $\vec{v}^0(x)$, $T^0(x) \in W_p^{\ell}(\mathbb{R}^3)$, then the following opriori estimates for the solutions of the Cauchy problem* (2′), (3) *take place:*

$$\|\vec{v}\|_{W^{k+1,\ell}_{p,t,x}(\mathbb{R}^4_H)} + \|T\|_{W^{k+1,\ell}_{p,t,x}(\mathbb{R}^4_H)} + \|\nabla_x P\|_{W^{k,\ell}_{p,t,x}(\mathbb{R}^4_H)}$$
$$\leq C_H(\ell,p,k)\ [\|\vec{v}^0\|_{W^{\ell}_p(\mathbb{R}^3)}\ \|T^0\|_{W^{\ell}_p(\mathbb{R}^3)}\ ],$$

where $\mathbb{R}^4_H = \{(x,t) : x \in \mathbb{R}^3, 0 \leq t \leq H\}$.

In addition, if $\vec{F} \in W^{k+1,\ell}_{p,t,x}(\mathbb{R}^4_H)$, then for the solutions of (2), (3), we will have;

$$\|\vec{v}\|_{W^{k+1,\ell}_{p,t,x}(\mathbb{R}^4_H)} + \|T\|_{W^{k+1,\ell}_{p,t,x}(\mathbb{R}^4_H)} + \|\nabla_x P\|_{W^{k,\ell}_{p,t,x}(\mathbb{R}^4_H)}$$
$$\leq C_H(\ell,p,k)\ [\|\vec{v}^0\|_{W^{\ell}_p(\mathbb{R}^3)} + \|T^0\|_{W^{\ell}_p(\mathbb{R}^3)} + \|\vec{F}\|_{W^{k+\ell}_{p,t,x}(\mathbb{R}^4_H)}\ ]$$

4. Asymptotic Behavior of Solution

When we consider a Cauchy problem, that is, large volumes of rotating fluids, there arise the problems of determining the behavior of solution as time $t \to \infty$. It is very important, even in numerical methods, to study the behavior of the solution for large time. In classical problems of mathematical physics (for example, the heat conduction equation), the question of whether a solution of the Cauchy problem for a homogeneous equation with smooth finite data tends to zero as $t \to \infty$, and at what rate, is usually simply solved. In the case of the system (2′), this question is not so simple because of the fact that the system contains a constantly operating

Coriolis term and therefore the equation of the damping of solution as $t \to \infty$ for the homogeneous system (2′) even when the initial data are sufficiently smooth and well decreasing as $|x| \to \infty$, requires further investigation. To obtain the asymptotic expansion of the solution, the kernels (4) are expressed in terms of Bessel's functions as follows:

$$
(7)\quad \begin{cases}
K_1(x,t) = \frac{1}{4\pi r}\left[J_0\left(\alpha\left(\frac{\rho}{r}\right)t\right) - \beta t \int\limits_0^t J_0\left(\alpha\left(\frac{\rho}{r}\right)\eta\right) \frac{J_1(\beta\sqrt{t^2-\eta^2})}{\sqrt{t^2-\eta^2}}\, d\eta \right] \\
K_2(x,t) = \frac{1}{4\pi r} \int\limits_0^t J_0\left(\alpha\left(\frac{\rho}{r}\right)\eta\right) J_0\left(\sqrt{t^2-\eta^2}\right) d\eta \\
K_3(x,t) = \frac{1}{4\pi^2 r} \int\limits_0^{\pi/2} \frac{1}{\alpha^2\left(\frac{\rho}{r}\right)^2 \sin^2\psi + \beta^2} \left[1 - \cos\left\{\alpha\left(\frac{\rho}{r}\right)\sin\psi t\right\} \right. \\
\qquad\qquad \left. + \beta t \int\limits_0^t \cos\left[\alpha\left(\frac{\rho}{r}\right)\eta\sin\psi\right] \frac{J_1(\beta\sqrt{t^2-\eta^2})}{\sqrt{t^2-\eta^2}}\, d\eta \right] d\psi.
\end{cases}
$$

Then, in investigating the convolutions in the solution's representation, change of variables, integration by parts, Chebyshev polynomials and their properties are used repeatedly. The integrals are approximated, mainly, by the method of stationary phase [3] and a Watson type lemma proved by Fedoryk is also used.

Before stating the theorem on asymptotics, we need to introduce the following condition:

Condition A. The initial data $\vec{v}^0(x)$ is said to satisfy condition A, if $\exists$ a positive constant C_3 such that $\forall$ multi-index β, $2 \le |\beta| \le 2\ell + 4$.

$$\int\limits_{\mathbb{R}^3} (1+|x|)^{|\beta|-2}\, |\mathcal{D}_x^\beta \vec{v}^0(x)|\, dx \le C_3$$

where ℓ is some given positive integer.

Theorem 4. *If* $\omega^2 = \sigma\gamma$, *solution of* (2′), (3) *is periodic in t. In case* $\omega^2 = \sigma\gamma$, *for initial data* $\vec{v}^0(x)$ *and* $T^0(x)$ *from* $C^{2\ell+4}(\mathbb{R}^3) \cap W_p^\ell(\mathbb{R}^3)$, *satisfying condition A, the solution of* (2′), (3) *satisfies the following properties*:

1. The components v_1, v_2, P and T stabilise at a rate $(\ln t)t^{-1/2}$ as $t \to \infty$ to some functions $v_1^*(x)$. $v_2^*(x)$. $P^*(x)$ and $T^*(x)$, respectively, which are determined completely by the prescribed data.
2. The components v_3 vanishes at a rate $t^{-1/2}$, as $t \to \infty$ at an arbitrary compact $K \subset \mathbb{R}^3$.

Here the functions $v_1^*(x)$. $v_2^*(x)$. $P^*(x)$ and $T^*(x)$ and $T^*(x)$ are given by

$$v_1^*(x) = \frac{\sqrt{\sigma\gamma}}{4\pi} \int_{\mathbf{R}^3} \left\{ \frac{\partial^2 v_1^0(x-y)}{\partial x_2^2} - \frac{\partial^2 v_2^0(x-y)}{\partial x_1 \partial x_2} + \frac{\omega}{\gamma} \frac{\partial^2 v_1^0(x-y)}{\partial x_1 \partial x_3} \right\} \frac{dy}{\sqrt{\omega^2 \rho^2 + \sigma\gamma\, y_3^2}}$$

$$v_1^*(x) = \frac{\sqrt{\sigma\gamma}}{4\pi} \int_{\mathbf{R}^3} \left\{ \frac{\partial^2 v_2^0(x-y)}{\partial x_1^2} - \frac{\partial^2 v_1^0(x-y)}{\partial x_1 \partial x_2} + \frac{\omega}{\gamma} \frac{\partial^2 T^0(x-y)}{\partial x_1 \partial x_3} \right\} \frac{dy}{\sqrt{\omega^2 \rho^2 + \sigma\gamma\, y_3^2}}$$

$$P^*(x) = \frac{\omega}{4\pi\sqrt{\sigma\gamma}} \int_{\mathbf{R}^3} \left\{ \frac{\partial^2 v_1^0(x-y)}{\partial x_2} - \frac{\partial^2 v_2^0(x-y)}{\partial x_1} + \frac{\omega}{\gamma} \frac{\partial^2 T^0(x-y)}{\partial x_2} \right\} \frac{dy}{\sqrt{\omega^2 \rho^2 + \sigma\gamma\, y_3^2}}$$

$$v_1^*(x) = \frac{\sqrt{\sigma\gamma}}{4\pi\sigma} \int_{\mathbf{R}^3} \left\{ \frac{\partial^2 v_1^0(x-y)}{\partial x_2^2\, \partial x_3} - \frac{\partial^2 v_2^0(x-y)}{\partial x_1 \partial x_3} + \frac{\omega}{\gamma} \frac{\partial^2 T^0(x-y)}{\partial x_3^2} \right\} \frac{dy}{\sqrt{\omega^2 \rho^2 + \sigma\gamma\, y_3^2}}$$

5. Concluding Remarks

For any physical problem to be well-posed, that is meaningful, the existence, uniqueness and stability of its solution are required. Theorems 1 to 4 show that our problems is well-posed. The results obtained for such a problem can be applied in various fields, such as, the science of atmosphere and oceans, the weather forecasting, theory of hydro-nuclear reactors, etc.

Due to the advent of powerful computers and advanced numerical methods, many problems now can be solved numerically. But at the same time, analytical methods are not to be underestimated as the analytical and numerical solutions have to complements each other. Explicit solution and its asymptotics obtained in this work give the possibility to determine the initial data effectively in each step of calculation, which minimizes the time required to solve the problem numerically.

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