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On the compact support of solutions to a nonlinear long internal waves model

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Abstract: We use complex analysis techniques to prove that, if a sufficiently regular solution to a model that governs the unidirectional propagation of long internal waves in a rotating homogeneous incompressible fluid is supported compactly in a non trivial time interval then it vanishes identically.

Key words: Dispersive equations; unique continuation property; smooth solution; compact support.

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1. Introduction:

In this work we are interested in studying the following initial value problem (IVP):

$$(1.1)\qquad \begin{cases} (u_t - \beta u_{xxx} + (u^2)_x)_x - \gamma u = 0, & x, t \in \mathbb{R} \\ u(x,0) = u_0(x), \end{cases}$$

where $u = u(x,t)$ is a real valued function and γ, β are constants. This model was introduced by Ostrovsky in [12] which describes the propagation of weakly nonlinear long surface and internal waves of small amplitude in a rotating homogeneous incompressible fluid. In literature, this model is also called as Ostrovsky equation. The parameters $\gamma > 0$ and β describe the effect of rotation and type of dispersion respectively. The value $\beta = -1$ describes negative dispersion for surface and internal waves in the Ocean and Surface waves in a shallow channel with uneven bottom. The value $\beta = 1$ describes positive dispersion for capillary waves on the liquid surface or for magneto-acoustic oblique waves in plasma [1], [5], [6].

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Several authors have studied this model in recent literature, see for example [1],[5],[6],[11],[17] and references there in. In particular, Cauchy problem associated with (1.1) has been studied in [17].

In this work we are concerned about the unique continuation property (UCP) for the model (1.1). There are various forms of UCP in the literature, see for example [2],[8],[9],[10],[15]and references there in. The following is the definition of UCP given in [15], where the first result of UCP for a dispersive model is proved.

Definition [15]. Let L be an evolution operator acting on functions defined on some connected open set Ω of $\mathbb{R}^n \times \mathbb{R}_t$. The operator L is said to have unique continuation property if every solution u of $Lu = 0$ that vanishes on some nonempty open set $\mathcal{O} \subset \Omega$ vanishes in the horizontal component of $\mathcal{O}$ in Ω.

Much effort has been used in studying UCP for various models in recent literature, for example [2],[3],[4],[7],[8],[9],[10],[13],[14],[15],[16]and [18] are just few to mention. In most cases Carleman type estimates are used to prove UCP. Recently Bourgain in [2] introduced a new method based on complex analysis to prove UCP for dispersive models. Although, by using Paley-Wiener theorem, the UCP for linear dispersive models, with this method, is almost immediate, the same is not so simple when one considers full nonlinear model. Some extra and technical efforts are necessary to address the case of nonlinear model. In this work we use method in [2] to prove that, if a sufficiently smooth solution to the IVP (1.1) it supported compactly in a non trivial time interval then it vanishes identically. In some sense it is a weak version of the UCP given in the above definition. Due to technical reason (see proof of Theorem 1.1, below) we consider the negative dispersion case i.e. $\beta = -1$, in (1.1). The main result of this work reads as follows:

Theorem 1.1: Let $u \in C(\mathbb{R}, H^s(\mathbb{R}))$ *be a solution to the* IVP (1.1) *with* $s > 0$ *large enough. If there exists a non trivial time interval* $I = [-T, T]$ *such that for some* $B > 0$,

$$\operatorname{supp} u(t) \subseteq [-B, B], \ \forall \ t \in I,$$

then $u \equiv 0$.

To prove this theorem we write the IVP (1.1) as

$$\begin{cases} u_t - \beta u_{xxx} + (u^2)_x - \gamma D_x^{-1} u = 0, \\ u(x,0) = u_0(x). \end{cases} \tag{1.2}$$

Now, we use Duhamel's formula to write the IVP (1.2) in the equivalent integral form

$$u(t) = U(t)u_0 - \int_0^t U(t-t')(u^2)_x(t')dt'. \tag{1.3}$$

where $U(t)$ is the unitary group describing the solution to the linear problem

$$\begin{cases} u_t - \beta u_{xxx} - \gamma D_x^{-1} u = 0 \\ u(x,0) = u_0(x), \end{cases} \tag{1.4}$$

and is given by

(1.5) $$U(t)u_0(x) = \frac{1}{\sqrt{2\pi}} \int_{\mathrm{R}} e^{i(x\xi - (\beta\xi^3 + \frac{\gamma}{\xi})t)} \hat{u}_0(\xi) d\xi.$$

Note that following are the conserved quantities satisfied by the flow of (1.1):

(1.6) $$\int_{\mathrm{R}} |u(x,t)|^2 dx. \qquad \text{(momentum)}$$

(1.7) $$\int_{\mathrm{R}} \beta\, u_x^2 + \tfrac{\gamma}{2}\left(D_x^{-1}u\right)^2 + \tfrac{1}{3}\, u^3 dx. \qquad \text{(energy)}$$

We organise this article as follows. We establish some preliminary estimates in section 2 and in section 3 we supply the proof of the main result of this work, Theorem 1.1.

Now we introduce some notations that will be used throughout this article. The Fourier transform of a function f denoted by $\hat{f}$ is defined as

$$\hat{f}(\xi) = \frac{1}{\sqrt{2\pi}} \int_{\mathrm{R}} e^{-ix\xi} f(x) dx.$$

We use H^s to denote L^2-based Sobolev space with index s. The various constants whose exact values are immaterial will be denoted by c. We use supp f to denote support of a function f and $f * g$ to denote the usual convolution product of f & g. Also, we use the notation $A \lesssim B$, if there exists a constant $c > 0$ such that $A \le cB$.

2. Preliminary estimates

In this section we record some preliminary estimates that are essential in the proof of our main result. The details of the proof of these estimates can be found in [2] and the author's previous works [13] & [14]. For the shake of clearness we just sketch the idea of the proofs.

Let us start by recording the following result.

Lemma 2.1: *Let $u \in C([-T,T]; H^s(\mathrm{R}))$ be a sufficiently smooth solution to the* IVP (1.1). *If for some $B > 0$, supp $u(t) \subseteq [-B, B]$, then for all $\xi, \theta \in \mathrm{R}$, we have*

(2.1) $$|\hat{u}(t)(\xi + i\theta)| \lesssim e^{C|\theta|B}.$$

Proof: The proof follows by using the Cauchy-Schwarz inequality and the conservation law (1.6). The argument is similar to the 2-dimensional case presented in [13] & [14].

Now we define

(2.2) $$u^*(\xi) = \sup_{t \in I} |u(t)(\xi)|.$$

and

(2.3) $$m(\xi) = \sup_{\xi' \ge \xi} |u^*(\xi')|.$$

Considering the initial data $u(0)$ sufficiently smooth and taking into account the well-posedness theory for the IVP(1.1) (see for e.g., [17] we have the following result.

Lemma 2.2: *Let $u \in C([-T,T]; H^s(\mathbb{R}))$ be a sufficiently smooth solution to the* IVP (1.1) *with supp $u(t) \subseteq [-B, B]$, $\forall\, t \in I$, then for some constant B_1, we have*

(2.4) $$m(\xi) \le \frac{B_1}{1+|\xi|^4}$$

Proof: The proof follows by using Cauchy-Schwarz inequality, conservation law (1.6) and well-posedness theory with the similar argument in the author's previous works [13] & [14].

Proposition 2.3: Let $u(t)$ be compactly supported and suppose that there exists $t \in I$ with $u(t) \neq 0$. Then there exists a number $C > 0$ such that for any large number $Q > 0$ there are arbitrary large ξ-values such that

(2.5) $$m(\xi) > C\,(m * m)\,(\xi)$$

and

(2.6) $$m(\xi) > e^{-\frac{|\xi|}{Q}}$$

Proof: The main ingredient in the proof of this Lemma is the estimate (2.4) in Lemma 2.2. The detail argument is similar to the one given in the proof of lemma in page 440 in [2], so we omit it.

Now, using the definition of $m(\xi)$ and Proposition 2.3 we can choose ξ large enough and $t_1 \in I$ such that

(2.7) $$|\hat{u}(t_1)(\xi)| = u^*(\xi) = m(\xi) > C\,(m * m)\,(\xi) + e^{\frac{-|\xi|}{Q}}.$$

In what follows we prove some derivative estimates for entire function. We start with the following result whose proof is given in [2].

Lemma 2.4: Let $\phi : \mathbb{C} \to \mathbb{C}$ *be an entire function which it bounded and integrable on the real axis and satisfies*

$$|\phi(\xi + i\theta)| \le e^{|\theta| B}, \qquad \xi, \theta \in \mathbb{R}.$$

Then, for $\xi_1 \in \mathbb{R}^+$ we have

(2.8) $$|\phi'(\xi)| \le B\Big(\sup_{\xi' \ge \xi_1} |\phi(\xi')|\Big)\Big[1 + \Big|\log\Big(\sup_{\xi' \ge \xi_1} |\phi(\xi')|\Big)\Big|\Big].$$

Corollary 2.5. Let $\theta \in \mathbb{R}$ be such that

(2.9) $$|\theta| \le B^{-1}\Big[1 + \Big|\log\Big(\sup_{\xi' \ge \xi_1 > 0} |\phi(\xi')|\Big)\Big|\Big]^{-1}$$

Then,

(2.10) $$\sup_{\xi' \ge \xi_1} |\phi(\xi' + i\theta)| \le 2 \sup_{\xi' \ge \xi_1} |\phi(\xi')|,$$

and

(2.11) $$\sup_{\xi'\geq\xi_1} |\phi(\xi'+i\theta)| \leq B\left(\sup_{\xi'\geq\xi_1}|\phi(\xi')|\right)\left[1+\left|\log\left(\sup_{\xi'\geq\xi_1}|\phi(\xi')|\right)\right|\right].$$

Proof: Detailed proof of this corollary can be found in Corollary 2.9 in [2]. So we omit it.

Now we state the last result of this section whose proof can be found in the author's previous works [13] & [14].

Corollary 2.6. Let $t \in I$, $\phi(z) = \widehat{u(t)}(z)$, θ be as in Corollary 2.5 and $m(\xi)$ be as in definition (2.3). Then for $|\theta'| \leq |\theta|$ fixed, we have

(2.12) $$|\phi'(\xi-\xi'+i\theta')| \leq B\left[m(\xi)+m(\xi-\xi')\right]\left[1+|\log m(\xi)|\right]$$

3. Proof of the main result

Now we are in position to supply proof of the main result of this work. The main idea in the proof is similar to the one employed in [2], [13] and [14], but the structure of the Fourier symbol associated with the linear part of the IVP (1.1) demands special attention and some basic modifications.

Proof of Theorem 1.1: We prove this theorem by contradiction.
If possible, suppose that there is some $t \in I$ such that $u(t) \neq 0$. Now our goal is to use the estimates derived in the previous section to arrive at a contradiction.

Let $t_1, t_2 \in I$ with t_1 as in (2.7). Using Duhamel's formula, we have

(3.1) $$u(t_2) = U(t_2-t_1)u(t_1) - c\int_{t_1}^{t_2} U(t_2-t')(u^2)_x(t')dt'$$

Taking Fourier transform in the space variable in (3.1) we get

(3.2) $$\begin{aligned}\widehat{u(t_2)}(\xi) &= e^{-i(t_2-t_1)(\beta\xi^3+\frac{\gamma}{\xi})}\,\hat{u}(t_1)(\xi)\,dt - \\ &\quad - ci\xi\int_{t_1}^{t_2} e^{-i(t_2-t')(\beta\xi^3+\frac{\gamma}{\xi})}\,\widehat{u^2(t')}(\xi)\,dt'.\end{aligned}$$

Let $\Delta t = t_2 - t_1$ and make a change of variable $s = t' - t_1$ to obtain,

(3.3) $$\begin{aligned}\widehat{u(t_2)}(\xi) &= e^{-i\Delta t(\beta\xi^3+\frac{\gamma}{\xi})}\,\widehat{u(t_1)}(\xi) - \\ &\quad - ci\xi\int_0^{\Delta t} e^{-(i\Delta t-s)(\beta\xi^3+\frac{\gamma}{\xi})}\,\widehat{u^2(t_1+s)}(\xi)\,ds \\ &= e^{-i\Delta t(\beta\xi^3+\frac{\gamma}{\xi})}\left[\widehat{u(t_1)}(\xi) - ci\xi\int_0^{\Delta t} e^{is(\beta\xi^3+\frac{\gamma}{\xi})}\,\widehat{u^2(t_1+s)}(\xi)\right].\end{aligned}$$

Since $u(t)$, $t \in I$ is compactly supported, by Paley-Wiener theorem, $u(t_2)(\xi)$ has analytic continuation in $\mathbb{C}$ and we have

$$\widehat{u(t_2)}(\xi+i\theta)=e^{-i\Delta t\left\{\beta(\xi+i\theta)^3+\frac{\gamma}{\xi+i\theta}\right\}}\Big[\widehat{u(t_1)}(\xi+i\theta)-$$

$$-ci(\xi+i\theta)\int_0^{\Delta t}e^{is\left\{\beta(\xi+i\theta)^3+\frac{\gamma}{\xi+i\theta}\right\}}\widehat{u^2(s+t_1)}(\xi+i\theta)\,ds\Big]. \tag{3.4}$$

Since,

$$\beta(\xi+i\theta)^3+\frac{\gamma}{\xi+i\theta}=\beta(\xi^3-3\xi\theta^2)+\frac{\gamma\xi}{\xi^2+\theta^2}+i\left(3\beta\xi^2\theta-\theta^3-\frac{\gamma\theta}{\xi^2+\theta^2}\right),$$

using Lemma 2.1, we obtain from (3.4)

$$ce^{-\Delta t\left(3\beta(\xi^2\theta-\theta^3)-\frac{\gamma\theta}{\xi^2+\theta^2}\right)}\geq|\widehat{u(t_1)}(\xi+i\theta)|-$$

$$-ci\,|\xi+i\theta|\int_0^{\Delta t}e^{-s\left(3\beta\xi^2\theta-\theta^3-\frac{\gamma\theta}{\xi^2+\theta^2}\right)}|\widehat{u^2(s+t_1)}(\xi+i\theta)|\,ds. \tag{3.5}$$

Now, let us select ξ very large and $\theta=\theta(\xi)$ such that $|\theta|\approx 0$. i.e.

$$\frac{1}{|\xi|}\ll|\theta|. \tag{3.6}$$

Also, let us choose sign of θ in such a way that

$$\theta\Delta t<0. \tag{3.7}$$

Now using these choices we get from (3.5)

$$ce^{-\Delta t\left(3\beta\xi^2\theta-\frac{\gamma\theta}{\xi^2+\theta^2}\right)}\geq|\widehat{u(t_1)}(\xi+i\theta)|-$$

$$-|\xi|\int_0^{\Delta t}e^{-s\left(3\beta\xi^2\theta-\frac{\gamma\theta}{\xi^2+\theta^2}\right)}|\widehat{u^2(s+t_1)}(\xi+i\theta)|\,ds. \tag{3.8}$$

Now considering the negative dispersion case, i.e., $\beta=-1$ and taking into account of (3.6) and (3.7), we obtain from (3.8).

$$ce^{-|\Delta t|\left(3\xi^2|\theta|+\frac{\gamma|\theta|}{\xi^2+\theta^2}\right)}\geq|\widehat{u(t_1)}(\xi+i\theta)|-$$

$$-|\xi|\int_0^{|\Delta t|}e^{-s\left(3\xi^2|\theta|+\frac{\gamma|\theta|}{\xi^2+\theta^2}\right)}|\widehat{u^2(t_1\pm s)}(\xi+i\theta)|\,ds. \tag{3.9}$$

where '+' sign corresponds to $\Delta t>0$ and '−' sign to $\Delta t<0$. From here onwards we consider the $\Delta t>0$ case only, the other case follows similarly. Since $e^{-x}<0$ for $x>0$ we can write the estimate (3.9) as,

$$ce^{\left(3\xi^2+\frac{\gamma}{\xi^2+\theta^2}\right)|\theta\Delta t|} \geq |\widehat{u(t_1)}(\xi+i\theta)| -$$

$$-|\xi| \int_0^{\Delta t} e^{-s\left(3\xi^2+\frac{\gamma}{\xi^2+\theta^2}\right)|\theta|} |\widehat{u^2(t_1+s)}(\xi+i\theta)|\,ds. \tag{3.10}$$

Finally we write the estimate (3.10) in the following way

$$ce^{-\left(3\xi^2|\theta|+\frac{\gamma|\theta|}{\xi^2+\theta^2}\right)|\theta\Delta t|} \geq |u(t_1)(\xi)| - |\xi| \int_0^{\Delta t} e^{-s\left(3\xi^2+\frac{\gamma}{\xi^2+\theta^2}\right)|\theta|} |\widehat{u^2(t_1+s)}(\xi)|\,ds -$$

$$-|\widehat{u(t_1)}(\xi+i\theta) - \widehat{u(t_1)}(\xi)| -$$

$$-|\xi| \int_0^{\Delta t} e^{-s\left(3\xi^2+\frac{\gamma}{\xi^2+\theta^2}\right)|\theta|} |\widehat{u^2(t_1+s)}(\xi+i\theta) -$$

$$-\widehat{u^2(t_1+s)}(\xi)|\,ds. \tag{3.11}$$

$$:= I_1, - I_2 - I_3$$

In sequel we use the preliminary estimates from the previous section to get appropriate estimates for I_1, I_2 and I_3 to arrive at a contradiction in (3.11).

Now we use definition of $u^*(\xi)$ and the estimate (2.5) to obtain

$$|\xi| \int_0^{\Delta t} e^{-s\left(3\xi^2+\frac{\gamma}{\xi^2+\theta^2}\right)|\theta|} |\widehat{u(t_1+s)}| * |\widehat{u(t_1+s)}|(\xi)\,ds \leq$$

$$\leq |\xi|(u^* * u^*)(\xi) \int_0^{\Delta t} e^{-s\left(3\xi^2+\frac{\gamma}{\xi^2+\theta^2}\right)|\theta|}\,ds$$

$$\leq |\xi|(m*m)(\xi)\,\frac{1-e^{-\Delta t\left(3\xi^2+\frac{\gamma}{\xi^2+\theta^2}\right)|\theta|}}{\left(3\xi^2+\frac{\gamma}{\xi^2+\theta^2}\right)|\theta|}$$

$$\leq \frac{|\xi|(m*m)(\xi)}{3\xi^2|\theta|}$$

$$\leq \frac{m(\xi)}{3|\xi\theta|}.$$

Therefore we get,

$$I_1 \geq m(\xi) - \frac{m(\xi)}{3|\xi\theta|} \geq \frac{m(\xi)}{3}. \tag{3.12}$$

To obtain estimate for I_2 we define $|\phi(z)| = \widehat{u(t_1)}(z)$, for $z \in \mathbb{C}$. Using (2.7) we get,

$$(3.13)\qquad |\theta(z) = |\widehat{u(t_1)}(\xi)| = \sup_{|\xi'|\geq|\xi|} |\phi(\xi')| = m(\xi).$$

Now, choose θ such that

$$(3.14)\qquad |\theta| \leq B^{-1}[1+|\log m(\xi)|]^{-1}.$$

Using Corollary 2.5 we obtain

$$\begin{aligned} I_2 &\leq |\theta| \sup_{|\xi'|\geq|\xi|} |\partial \widehat{u(t_1)}(\xi'+i\theta)|. \\ &\leq |\theta|\, B m(\xi)[1+|\log m(\xi)|]^{-1} \\ &\leq m(\xi) \leq \tfrac{1}{15} m(\xi). \end{aligned}$$

Finally to get estimate for I_3 we use Proposition 2.3, Corollary 2.6 and θ as in (3.14) to obtain

$$\begin{aligned} &|\widehat{u^2(t_1+s)}(\xi+i\theta) - \widehat{u^2(t_1+s)}(\xi)| \leq \\ &\leq \int_{\mathbb{R}} |\widehat{u(t_1+s)}(\xi-\xi'+i\theta) - \widehat{u(t_1+s)}(\xi-\xi')||\widehat{u(t_1+s)}(\xi')|\, d\xi' \\ &\leq |\theta| \int_{\mathbb{R}} \sup_{|\xi'|\leq|\xi|} |\partial \widehat{u(t_1+s)}(\xi-\xi'+i\theta')|\, m(\xi')\, d\xi' \\ &\leq \int_{\mathbb{R}} [m(\xi) - m(\xi-\xi')]\, m(\xi')\, d\xi' \\ &\leq m(\xi)\, c_2 + (m * m)(\xi) \\ &\leq m(\xi)(c_2 + c^{-1}) \leq m(\xi). \end{aligned}$$

Therefore,

$$\begin{aligned} I_3 &\leq |\xi|\, m(\xi) \int_0^{\Delta t} e^{-s\left(3\xi^2 + \frac{\gamma}{\xi^2+\theta^2}\right)|\theta|}\, ds \\ &= |\xi|\, m(\xi)\, \frac{1 - e^{-\Delta t\left(3\xi^2 + \frac{\gamma}{\xi^2+\theta^2}\right)|\theta|}}{\left(3\xi^2 + \frac{\gamma}{\xi^2+\theta^2}\right)|\theta|} \\ &\leq \frac{|\xi|\, m(\xi)}{3\xi^2|\theta|} \\ &\leq \frac{m(\xi)}{3|\xi\theta|} \leq \tfrac{1}{15} m(\xi). \end{aligned}$$

Now, using (3.12), (3.15) and (3.16) in (3.11) and using the estimate (2.6) one gets.

$$(3.17)\qquad e^{-\left(3\xi^2 + \frac{\gamma}{\xi^2+\theta^2}\right)|\theta\Delta t|} \geq \frac{m(\xi)}{3} - \frac{m(\xi)}{15} - \frac{m(\xi)}{15} = \frac{1}{3} m(\xi) \geq e^{\frac{-|\xi|}{Q}}.$$

On the other hand, with the choice of ξ and θ we have

$$e^{-\left(3\xi^2+\frac{\gamma}{\xi^2+\theta^2}\right)|\theta||\Delta t|} \leq e^{-|\xi||\Delta t|}. \tag{3.18}$$

Now from (3.17) and (3.18) we obtain

$$e^{-|\xi||\Delta t|} \geq e^{\frac{-|\xi|}{Q}}, \tag{3.19}$$

which is false for $|\xi|$ large if we choose Q large enough such that $\frac{1}{Q} < |\Delta t|$. This contradiction completes the proof of the theorem.

REFERENCES

[1] Benilov E. S. ; *On the surface waves in a shallow channel with uneven bottom*, Stud. Appl. Math. **87** (1992) 1–14.

[2] Bourgain J. ; *On the Compactness of the support of solutions of dispersive equations*, IMRN, 9(1997) 437–447.

[3] Carvajal X., Panthee M.; *Unique continuation property for a higher order nonlinear Schrödinger equation*, J. Math. Anal. Appl., **303** (2005) 188–207.

[4] Carvajal X., Panthee M., *On uniqueness of solution for a nonlinear Schrödinger Airy equation*, to appear in Nonlinear Analysis, TMA.

[5] Galkin V.N., Stepanyants Y.A., ; *On the existence of stationary solitary waves in a rotating* fluid, J. Appl. Math. Mech., **55**(6) (1991) 939–943.

[6] Gilman O.A., Grimshaw R., Stepanyants Y.A., ; *Approximate and numerical solutions of the stationary Ostrovsky equation*, Stud. Apl. Math., **95**(1995) 115–126.

[7] Iório Jr. R.J.; *Unique continuation principles for some equations of Benzamin-Ono type, preprint*, IMPA (2002).

[8] Kenig C.E., Ponce G, Vega L.; *On the support of solutions to the generalized KdV equation*, Ann I.H. Poincare, **19** (2) (2002) 191–208.

[9] Kenig C.E., Ponce G, Vega L.; *On unique continuation for nonlinear Schrödinger equation*, Comm. Pure Appl. Math., **56**(2003) 1247–1262.

[10] Kenig C.E., Ponce G, Vega L.; *On unique continuation of solutions to the generalized KdV equation*, Math. Res. Lett., **10** (2003) 833–846.

[11] Linares F., Milaès A. ; *A note on solutions to a model for long internal waves in a rotating fluid*, Preprint (2005).

[12] Ostrovsky L. A.; *Nonlinear internal waves in a rotàting Ocean,* Okeanlogia, **18**(2) (1978) 181–191.

[13] Panthee M., *A note on the unique continuation property for Zakharov-Kuznetsov equation*, Nonlinear Analysis; TMA, **59** (2004) 425–438.

[14] Panthee M.; *Unique continuation property for the Kadomtsev-Petviashvili* (KP-II) *equation*, Electronic J. Diff. Equations, **2005** (2005) 1–12.

[15] Saut J-C., Scheurer B.; *Unique continuation for some evolution equations*, J. Diff Equations, **66**(1987) 118-139.

[16] Tataru D., ; *Carleman type estimates and unique continuation for the Schrödinger equation*, Diff. and Int. Equations, **8**(4) (1995) 901–905.

[17] Valamov V., Liu Y.; *Cauchy problem for the Ostrovsky equation*, Discrete & Cont. Dyn. Syst. **10**(3) (2004) 731–753.

[18] Zhang B.Y.; *Unique continuation for the Korteweg-de Vries equation*, SIAM J. Math. Anal., **23**(1992) 55–71.

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