The Nepali Math. Sc. Report
Vol. 29, No. 1 & 2, 2009

Operation Approaches on Fuzzy Pre-Open Sets

M. SUDHA, E. ROJA & M.K. UMA
Department of Mathematics,
Sri Sarada College For Women,
Salem – 636 016.
Tamilnadu.
India.

Abstract: In this paper, the concepts of an operation γ on a family of fuzzy pre-open sets in a fuzzy topological spaces (X, T) is introduced. Using the operation γ on FPO (X) the concepts of fuzzy pre-γ-open sets, fuzzy pre-γ-border, fuzzy pre-γ-frontier, fuzzy pre-(γ, β)-continuous mappings, fuzzy pre-γ-normal spaces and fuzzy pre-γ-compact spaces are introduced. Some interesting properties and characterizations of them are investigated. Further, fuzzy pre-γ-R_o and fuzzy pre-γ-T_i (i = 0, 1/2, 1, 2) spaces are introduced and interrelations among the spaces are discussed with relevant examples.

Key Words

Fuzzy pre-γ-open set, fuzzy pre-γ-border, fuzzy pre-γ-frontier, fuzzy pre-(γ, β)-continuous mapping, fuzzy pre-γ-normal space, fuzzy pre-γ-compact space, fuzzy pre-γ-R_o space, fuzzy pre- γ-T_i (i = 0, 1/2 , 1, 2) space.

2000 Mathematics subject classification : 54A40-03E72.

1. INTRODUCTION AND PRELIMINARIES

The concept of fuzzy sets has invaded almost all branches of mathematics since the introduction of the concept by Zadeh [11]. Fuzzy sets have applications in many fields such as information [7] and control [8]. The theory of fuzzy topological spaces was introduced and developed by Chang [3]. The concept of

fuzzy pre-open sets and fuzzy pre–closed sets were introduced by Singal and Prakash [6]. The concept of fuzzy pre-continuity was introduced by Bin Shahna [1] and was studied by Uma, Roja and Balasubramanian [10]. By using the concepts of semi-γ-open sets introduced by Sai Sundara Krishnan, Ganster and Balachandran [4] and that of g-border and g-frontier introduced by Caldas, Jafari and Noiri [2], the concepts of fuzzy pre-γ-open set, fuzzy pre-γ-border, pre-γ-frontier, fuzzy pre-(γ, β)-continuous mappings, fuzzy pre-γ-normal spaces, fuzzy pre-γ-compact spaces, fuzzy pre-γ-T_i $(i = 0, 1, 2)$ spaces and fuzzy pre-γ-R_o space are introduced and interrelations among the spaces are discussed with relevant examples.

Definition 1.1 [6]

Let (X, T) be a fuzzy topological space. A fuzzy set λ in (X, T) is said to be fuzzy pre-open if $\lambda \leq \text{int cl}(\lambda)$.

The complement of a fuzzy pre-open set is fuzzy pre-closed.

Definition 1.2 [4]

Let (X, T) be a fuzzy topological space. An operation γ on the topology T is a mapping from T into power set P(X) of X such that $V \subseteq V^{\gamma}$ for each $V \in T$, where V^{γ} denotes the value of γ at V. It is denoted by $\gamma : T \to P(X)$.

Definition 1.3 [4]

A subset A of a topological space is called a γ-open set of (X, T) if for each $x \in A$ there exists an open set U such that $x \in U$ and $U^{\gamma} \subseteq A$. The complement of a γ-open set is said to be γ-closed.

Notation 1.1 [4]

SO(X) denotes the family of all semi-open sets of (X, T).

Definition 1.4 [4]

Let (X, T) be a topological space. An operation γ on the SO(X) is a mapping from SO(X) into a power set P(X) of X such that $V \subseteq V^{\gamma}$ for each $V \in SO(X)$ and V^{γ} denotes the value of γ at V. It is denoted by $\gamma : SO(X) \to P(X)$.

Definition 1.5 [4]

Let (X, T) be a topological space and γ be an operation on SO(X). Then a subset A of X is said to be a semi-γ-open set if for each $x \in A$, there exists a semi-open set U such that $x \in U$ and $U^{\gamma} \subseteq A$. Also $SO(X)_{\gamma}$ denotes the family of semi-γ-open sets in X.

Definition 1.6 [2]

Let (X, T) be a topological space. For a subset A of (X, T), $b_g(A) = A - int_g(A)$ is said to be the g-border of A where $int_g(A)$ is the set of all g-interior points of A.

Definition 1.7 [2]

Let (X, T) be a topological space. For a subset A of (X, T), $Fr_g(A) = cl_g(A) - int_g(A)$ is said to be the g-frontier of A.

Definition 1.8 [9]

A topological space (X, T) is said to be a fuzzy pre-$T_{1/2}$ space if every gfpre-closed set in (X, T) is fuzzy closed in (X, T).

Definition 1.9 [5]

A fuzzy set λ is quasi-coincident with a fuzzy set μ, denoted by $\lambda\, q\, \mu$, if there exists $x \in X$ such that $\lambda(x) + \mu(x) > 1$. Otherwise $\lambda \not{q}\, \mu$.

2. FUZZY PRE-γ-OPEN SETS

Definition 2.1

Let (X, T) be a fuzzy topological space. Let $\gamma : I^X \to T$ be an operation such that $\lambda^\gamma = \wedge\, \mu$ where $\lambda \le \mu$, for each fuzzy open set μ in (X, T), $\lambda \in I^X$ and λ^γ denotes the value of γ at λ. That is, $\lambda^\gamma = \gamma(\lambda)$.

Definition 2.2

Let (X, T) be a fuzzy topological space. Let $\gamma : I^X \to T$ be an operation. A fuzzy set δ is said to be fuzzy-γ-open if for a fuzzy set α with $\alpha \le \delta$, there exists a fuzzy open set λ such that $\alpha \le \lambda$ and $\lambda^\gamma \le \delta$.

The complement of a fuzzy γ-open-set is fuzzy-γ-closed.

Definition 2.3

Let (X, T) be a fuzzy topological space. Let $\gamma : I^X \to T$ be an operation. For any fuzzy set λ, fuzzy-γ-interior of λ (briefly, γ-int (λ)) is defined as $\gamma\text{-int}(\lambda) = \vee\{\mu : \mu \le \lambda$ and μ is fuzzy-γ-open$\}$.

Definition 2.4

Let (X, T) be a fuzzy topological space. Let $\gamma : I^X \to T$ be an operation. For any fuzzy set λ, fuzzy-γ-closure of λ (briefly, γ-cl (λ)) is defined as $\gamma\text{-cl}(\lambda) = \wedge\{\mu : \mu \ge \lambda$ and μ is fuzzy-γ-closed$\}$.

Remark 2.1

γ-int $(1 - \lambda) = 1 - (\gamma\text{-cl}(\lambda))$.

Notation 2.1

FPO (X) denotes the family of all fuzzy pre-open sets of (X, T).

Definition 2.5

Let (X, T) be a fuzzy topological space. Let γ : FPO (X) $\rightarrow$ T be an operation such that $\lambda^{\gamma} = \wedge \mu$, where $\lambda \leq \mu$, for each fuzzy open set μ in (X, T) and $\lambda \in$ FPO (X).

Definition 2.6

Let (X, T) be a fuzzy topological space. Let γ be an operation on FPO (X). A fuzzy set δ is called fuzzy pre-γ-open if for a fuzzy set α with $\alpha \leq \delta$, there exists a fuzzy pre-open set λ such that $\alpha \leq \lambda$ and $\lambda^{\gamma} \leq \delta$.

The complement of a fuzzy pre-γ-open set is fuzzy pre-γ-closed.

Definition 2.7

Let (X, T) be a fuzzy topological space. Let γ be an operation on FPO (X). The fuzzy pre-γ-interior of δ (briefly, γ-fp int (δ)) is defined by γ-fp int (δ) = $\vee \{ \mu : \mu \leq \delta$ and μ is fuzzy pre-γ-open $\}$.

Definition 2.8

Let (X, T) be a fuzzy topological space. Let γ be an operation on FPO (X). The fuzzy pre-γ-closure of δ (briefly, γ-fp cl (δ)) is defined by γ-fp cl (δ) = $\wedge \{ \mu : \mu \geq \delta$ and μ is fuzzy pre-γ-closed $\}$.

Remark 2.2

γ-fp int$(1 - \delta) = 1 - (\gamma\text{-fp cl}(\delta))$.

Remark 2.3

Fuzzy pre-open set and fuzzy pre-γ-open set are independent notions.

Example 2.1

Let X = { a, b }. Define T = { 0, 1, λ_1, λ_2 } where $\lambda_1, \lambda_2 : X \rightarrow [0, 1]$ are defined as λ_1 (a) = 0.3, λ_1 (b) = 0.2, λ_2 (a) = 0.45, λ_2 (b) = 0.4. Let γ : FPO (X) $\rightarrow$ T be an operation. Let $\mu, \delta, \eta : X \rightarrow [0, 1]$ be defined as μ (a) = 0.4, μ (b) = 0.3, δ (a) = 0.45, δ (b) = 0.3, η (a) = 0.55, η (b) = 0.65. Now int cl (μ) $\geq \mu$. **Hence μ is fuzzy pre-open but not fuzzy pre-γ-open**. Now, for a fuzzy set α with $\alpha \leq \eta$, then $\alpha \leq \mu$ and $\mu^{\gamma} \leq \eta$. **Hence η is fuzzy pre-γ-open but not fuzzy pre-open.**

Proposition 2.1

Let (X, T) be a fuzzy topological space. Let λ and μ be any two fuzzy pre-γ-open sets in (X, T). Then $\lambda \vee \mu$ (resp. $\lambda \wedge \mu$) is also a fuzzy pre-γ-open set in (X, T).

Proposition 2.2

Let (X, T) be a fuzzy topological space. For any two fuzzy sets λ, μ, the following statements hold :

a. If λ is fuzzy-γ-open then λ is fuzzy pre-γ-open.

b. γ-int (λ) is fuzzy pre-γ-open.

c. γ-cl (λ) is fuzzy pre-γ-closed.

d. is fuzzy pre-γ-open iff $\lambda = \gamma$-fp int (λ).

e. is fuzzy pre-γ-closed iff $\lambda = \gamma$-fp cl (λ).

f. γ-int (λ) $\leq \gamma$-fp int (λ) $\leq \lambda \leq \gamma$-fp cl (λ) $\leq$ γ-cl (λ).

g. γ-cl (γ-fp cl (λ)) = γ-fp cl (λ).

h. γ-cl (γ-fp cl (λ)) = γ-fp cl (γ-cl (λ)) = γ-cl (λ).

i. (γ-fp int (λ)) $\wedge$ (γ-fp int (μ)) $\geq \gamma$-fp int ($\lambda \wedge \mu$).

j. (γ-fp int (λ)) $\vee$ (γ-fp int (μ)) $\leq \gamma$-fp int ($\lambda \vee \mu$).

Definition 2.9

Let (X, T) be a fuzzy topological space and let $\gamma : I^X \to T$ be an operation. For any fuzzy set λ, fuzzy-γ-border of λ (briefly, γ-fb (λ)) is defined as γ-fb (λ) = λ – (γ-int (λ)).

Definition 2.10

Let (X, T) be a fuzzy topological space and let γ be an operation on FPO (X) for any fuzzy set λ, fuzzy pre-γ-border of λ (briefly, γ-fpb (λ)) is defined as γ-fpb (λ) = λ – (γ-fp int (λ)).

Definition 2.11

Let (X, T) be a fuzzy topological space and let $\gamma : I^X \to T$ be an operation. For any fuzzy set λ, fuzzy-γ-frontier of λ (briefly, γ-f Fr (λ)) is defined as γ-f Fr (λ) = (γ-cl (λ)) – (γ-int (λ)).

Definition 2.12

Let (X, T) be a fuzzy topological space and let γ be an operation on FPO (X) . For any fuzzy set λ, fuzzy pre-γ-frontier of λ (briefly, γ-fp Fr (λ)) is defined as γ-fp Fr (λ) = (γ-fp cl (λ)) – (γ-fp int (λ)).

Proposition 2.3

Let (X, T) be a fuzzy topological space. For any two fuzzy sets λ, μ the following statements hold :

a. γ-fpb $(\lambda) \leq \gamma$-fp cl $(1 - \lambda)$.

b. γ-fpb $(\lambda \vee \mu) \leq (\gamma$-fpb $(\lambda)) \vee (\gamma$-fpb $(\mu))$.

c. γ-fpb $(\lambda \wedge \mu) \geq (\gamma$-fpb $(\lambda)) \wedge (\gamma$-fpb $(\mu))$.

d. $(\gamma$-int $(\lambda)) \vee (\gamma$-fb $(\lambda)) \geq \gamma$-int (λ).

e. $(\gamma$-int $(\lambda)) \wedge (\gamma$-fb $(\lambda)) \leq \gamma$-int (λ).

f. γ-fp Fr $(\lambda) = \gamma$-fp Fr $(1 - \lambda)$.

g. γ-fp Fr $(\gamma$-fp int $(\lambda)) \leq \gamma$-fp Fr (λ).

h. γ-fp Fr $(\gamma$-fp cl $(\lambda)) \leq \gamma$-fp Fr (λ).

i. $\lambda - (\gamma$-fp Fr $(\lambda)) \leq \gamma$-fp int (λ).

j. γ-fp Fr $(\lambda \vee \mu) \leq (\gamma$-fp Fr $(\lambda)) \vee (\gamma$-fp Fr $(\mu))$.

k. γ-fp Fr $(\lambda \wedge \mu) \geq (\gamma$-fp Fr $(\lambda)) \wedge (\gamma$-fp Fr $(\mu))$.

3. FUZZY PRE-γ-T_1 SPACES

Definition 3.1

A fuzzy topological space (X, T) is called

(a) a fuzzy pre-γ-T_o space iff for any two fuzzy sets λ, μ with $\lambda \not q \mu$, there exists a fuzzy pre-γ-open set δ such that $\lambda \leq \delta$, $\mu \not q \delta$ or $\mu \leq \delta$, $\lambda \not q \delta$.

(b) a fuzzy pre-γ-T_1 space iff for any two fuzzy sets λ, μ with $\lambda \not q \mu$, there exist fuzzy pre-γ- open sets δ, η such that either $\lambda \leq \delta$, $\mu \not q \delta$ or $\mu \leq \eta$, $\lambda \not q \eta$.

(c) a fuzzy pre-γ-T_2 space iff for any two fuzzy sets λ, μ with $\lambda \not q \mu$, there exist fuzzy pre-γ-open sets δ, η such that $\lambda \leq \delta$, $\mu \leq \eta$ and $\delta \not q \eta$.

(d) a fuzzy pre-γ-R_o space iff for any two fuzzy sets λ, μ, $\lambda \not q (\gamma$-fp cl $(\mu))$ implies that $\mu \not q (\gamma$-fp cl $(\lambda))$.

Definition 3.2

Let (X, T) be a fuzzy topological space and let γ be an operation on FPO(X). A fuzzy set λ is called fuzzy pre-γ-g closed if γ-fp cl $(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzy pre-γ-open.

The complement of a fuzzy pre-γ-gclosed set is fuzzy pre-γ-g open.

Definition 3.3

A fuzzy topological space (X, T) is called fuzzy pre-γ-$T_{1/2}$ space if every fuzzy pre-γ-g closed set is fuzzy pre-γ-closed.

Remark 3.1

From the above definitions we have the following implications.

fuzzy pre-γ-T_2 space $\Rightarrow$ fuzzy pre-γ-T_1 space $\Rightarrow$ fuzzy pre-γ-$T_{1/2}$space $\Rightarrow$ fuzzy pre-γ-T_o space.

The converse statements need not be true, as shown in the following examples.

Example 3.1

Let X = {a, b}. Define T = {0, 1, λ_1, λ_2, λ_3} where $\lambda_1, \lambda_2, \lambda_3 : X \to [0, 1]$ are defined as λ_1 (a) = 0.51, λ_1 (b) = 0.7, λ_2 (a) = 0.57, λ_2 (b) = 0.78, λ_3 (a) = 0.63, λ_3 (b) = 0.83. Let γ : FPO (X) $\to$ T be an operation. Let $\alpha, \mu, \delta, \eta : X \to [0, 1]$ be defined as α (a) = 0.3, α (b) = 0.4, μ (a) = 0.55, μ (b) = 0.75, δ (a) = 0.6, δ (b) = 0.8, η (a) = 0.65, η (b) = 0.85. Clearly μ is a fuzzy pre–open set. Now $\alpha \le \eta$, $\alpha \le \delta$ and $\alpha \le \mu$. Further $\mu^\gamma \le \delta$ and $\mu^\gamma \le \eta$. Therefore δ and η are fuzzy pre-γ-open sets. Let $\theta, \lambda : X \to [0, 1]$ be such that θ (a) = 0.3, θ (b) = 0.1, λ (a) = 0.2, λ (b) = 0. Then $\theta \not\le \lambda$. Further $\theta \le \delta$, $\lambda \not\le \delta$ and $\lambda \le \eta$, $\theta \not\le \eta$. **Hence (X, T) is a fuzzy pre-γ-T_1 space but not a fuzzy pre-γ-T_2 space.**

Example 3.2

Let X = {a, b}. Define T = { 0, 1, λ_1, λ_2, λ_3 } where $\lambda_1, \lambda_2, \lambda_3 : X \to [0, 1]$ are defined as λ_1 (a) = 0.5, λ_1 (b) = 0.6, λ_2 (a) = 0.7, λ_2 (b) = 0.75, λ_3 (a) = 0.8, λ_3 (b) = 0.9. Let γ : FPO (X) $\to$ T be an operation. The space (X, T) is a fuzzy pre-γ- $T_{1/2}$ space but not a fuzzy pre-γ-T_1 space.

Example 3.3

Let X = { a, b }. Define T = { 0, 1, λ_1, λ_2 } where

$\lambda_1, \lambda_2 : X \to [0, 1]$ are defined as λ_1 (a) = 0.3, λ_1 (b) = 0.2, λ_2 (a) = 0.45, λ_2 (b) = 0.4. Let γ : FPO (X) $\to$ T be an operation. Let $\alpha, \mu, \delta : X \to [0, 1]$ be defined as α (a) = 0.2, α (b) = 0.3, μ (a) = 0.4, μ (b) = 0.3, δ (a) 0.55, δ (b) = 0.65. Clearly μ is a fuzzy pre-open set. Now, $\alpha \le \delta$, $\alpha \le \mu$ and $\mu^\gamma \le \delta$. Therefore δ is a fuzzy pre-γ-open set. Let $\theta, \rho : X \to [0, 1]$be defined as θ (a) = 0.3, θ (b) = 0.4, ρ (a) = 0.4, ρ (b) = 0.2. Then $\theta \not\le \rho$. Now, $\theta \le \delta$ and $\rho \not\le \delta$. Hence (X, T) is a fuzzy pre-γ-T_o space. Let $\lambda : X \to [0, 1]$ be defined as λ (a) = 0.5, λ (b) = 0.45. Now, $\lambda \le \delta$ and γ-

fp cl(λ) $\leq \delta$. Therefore λ is a fuzzy pre-γ-g closed set. But not a fuzzy pre-γ-closed set. Hence (X, T) is not a fuzzy pre-γ-$T_{1/2}$ space.

Proposition 3.1

Let (X, T) be a fuzzy topological space . Then

(a) for all fuzzy pre-γ-open set λ in (X, T), λ q μ iff λ q (γ-fp cl (μ)), where μ is any fuzzy set in (X, T).

(b) δ q (γ-fp cl (λ)) iff λ q μ, for all fuzzy pre-γ-open set μ in (X, T), with $\delta \leq \mu$.

Proof:

(a) Let λ be a fuzzy pre-γ-open set such that λ q μ. Then since $\mu \leq \gamma$-fp cl (μ), λ q (γ-fp cl (μ)). Conversely let λ be a fuzzy pre-γ-open set in (X, T) such that $\lambda \not q\, \mu$. Then $\mu \leq 1 - \lambda$ and so γ-fp cl (μ) $\leq \gamma$-fp cl $(1 - \lambda) = 1 - \lambda$. Thus $\lambda \not q$ (γ-fp cl (μ)).

(b) Let δ q (γ-fp cl (λ)) and let μ be a fuzzy pre-γ-open set in (X, T) such that $\delta \leq \mu$. Then μ q (γ-fp cl (λ)). By (a), μ q λ for all fuzzy pre-γ-open set μ with $\delta \leq \mu$. Conversely suppose that $\delta \not q$ (γ-fp cl (λ)). Then $\delta \leq 1 - (\gamma$-fp cl (λ)). Let $\mu = 1 - (\gamma$-fp cl (λ)). Then μ is a fuzzy pre-γ-open set with $\delta \leq \mu$. Since $\lambda \leq \gamma$-fp cl (λ), $\mu = 1 - (\gamma$-fp cl (λ)) $\leq 1 - \lambda$. Therefore $\lambda \not q\, \mu$.

Proposition 3.2

Let (X, T) be a fuzzy topological space. For any two fuzzy sets δ, ρ in (X, T), the following statements are equivalent :

(a) (X, T) is a fuzzy pre-γ-R_o space.

(b) If $\delta \not q\, \lambda = \gamma$-fp cl ($\lambda$), where λ is any fuzzy set in (X, T), then there exists a fuzzy pre-γ-open set μ in (X, T), such that $\delta \not q\, \mu$ and $\lambda \leq \mu$.

(c) If $\delta \not q\, \lambda = \gamma$-fp cl ($\lambda$) then ($\gamma$-fp cl ($\delta$)) $\not q\, \lambda = \gamma$-fp cl ($\lambda$), where λ is any fuzzy set in (X, T).

(d) If $\delta \not q$ (γ-fp cl (ρ)) then (γ-fp cl (δ)) $\not q$ (γ-fp cl (ρ)).

Proof:

(a) $\Rightarrow$ (b) Let $\delta \not q\, \lambda = \gamma$-fp cl ($\lambda$). Since γ-fp cl (ρ) $\leq \gamma$-fp cl (λ), for each $\rho \leq \lambda$, $\delta \not q$ (γ-fp cl (ρ)). Then by (a), $\rho \not q$ (γ-fp cl (δ)). Then by (b) of Proposition 3.1, there exists a fuzzy pre-γ-open set η in (X, T), such that $\delta \not q\, \eta$ and $\rho \leq \eta$. Let $\mu = \vee \{\eta : \delta \not q\, \eta\}$. Then $\delta \not q\, \mu$ and $\lambda \leq \mu$, where μ is fuzzy pre-γ-open in (X, T).

(b)⇒ (c) Let $\delta \not\bar{q}\, \lambda = \gamma$-fp cl (λ). Then by (b), there exists a fuzzy pre-γ-open set μ in (X, T), such that $\delta \not\bar{q}\, \mu$ and $\lambda \leq \mu$. Since $\delta \not\bar{q}\, \mu$, $\delta \leq 1 - \mu$. Therefore γ-fp cl $(\delta) \leq \gamma$-fp cl $(1 - \mu) = 1 - \mu \leq 1 - \lambda$.

Hence (γ-fp cl (δ)) $\not\bar{q}\, \lambda = \gamma$-fp cl (λ).

(c)⇒ (d) Let $\delta \not\bar{q}$ (γ-fp cl (ρ)). since γ-fpcl (γ-fp cl (ρ)) = γ-fp cl (ρ), by (c), γ-fp cl $(\delta) \not\bar{q}$ (γ-fp cl (ρ)).

(d)⇒ (a) Let $\delta \not\bar{q}$ (γ - fp cl (ρ)). Then by (d),

(γ-fp cl (δ)) $\not\bar{q}$ (γ-fp cl (ρ)). Since $\rho \leq \gamma$-fp cl (ρ), $\rho \not\bar{q}$ (γ-fp cl (δ)). Hence (X, T) is a fuzzy pre-γ-R_o space.

4. FUZZY PRE - (γ, β)-CONTINUOUS MAPPINGS

Let (X, T), (Y, S) and (Z, R) be any three fuzzy topological spaces and let γ : FP0 (X) → T, β : FP0 (Y) → T and η : FP0 (Z) → T be operations on FP0 (X), FPO (Y) and FPO (Z) respectively.

Definition 4.1

Let f : (X, T) → (Y, S) be a mapping. Then

(a) f is called fuzzy pre-(γ, β)-continuous iff for each fuzzy pre-β-open set μ in (Y, S), $f^{-1}(\mu)$ is fuzzy pre-γ-open.

(b) f is called fuzzy pre-(γ, β)-closed iff for each fuzzy pre-γ-closed set λ in (X, T), f (λ) is fuzzy pre-β-closed.

(c) f is called fuzzy pre-(γ, β)-g continuous iff for each fuzzy pre-β-g closed set μ in (Y, S), $f^{-1}(\mu)$ is fuzzy pre-γ-g closed.

(d) f is called fuzzy pre-(γ, β)-g closed iff for each fuzzy pre-γ-g closed set λ in (X, T), f (λ) is fuzzy pre-β-g closed.

Proposition 4.1

A mapping f : (X, T) → (Y, S) is fuzzy pre-(γ, β)-continuous iff f (γ-fp cl (λ)) ≤ β-fp cl (f (λ)), for each fuzzy set λ in (X, T).

Proposition 4.2

A mapping f : (X , T) → (Y, S) is fuzzy pre-(γ, β)-continuous iff γ-fp cl $(f^{-1}(\lambda)) \leq f^{-1}$($\beta$-fp cl (λ)), for each fuzzy set λ in (Y, S).

Proposition 4.3

Let f : (X, T) → (Y, S) be a fuzzy pre-(γ, β)-continuous and g : (Y, S) → (Z, R) be a fuzzy pre-(β, η)-continuous mappings. Then gof : (X, T) → (Z, R) is fuzzy pre-(γ, η)-continuous.

Proposition 4.4

Let f : (X, T) → (Y, S) be a mapping. Then f is a fuzzy pre-(γ, β)-closed mapping iff β-fp cl (f (λ)) ≤ f (γ-fp cl (λ)), for each fuzzy set λ in (X, T).

Definition 4.2

Let f : (X, T) → (Y, S) be a bijective mapping. If both f and f^{-1} are fuzzy pre-(γ, β)-continuous, then f is called a fuzzy pre-(γ, β)-homeomorphism.

Proposition 4.5

Let f : (X, T) → (Y, S) be a bijective mapping. Then the following statements are equivalent:

(a) f is a fuzzy pre-(γ, β)-homeomorphism.

(b) f is a fuzzy pre-(γ, β)-continuous and fuzzy pre-(γ, β)-open mapping.

(c) f is a fuzzy pre-(γ, β)-continuous and fuzzy pre-(γ, β)-closed mapping.

(d) f (γ-fp cl (λ)) = β-fp cl (f (λ)), for each fuzzy set λ in (X, T)

Proposition 4.6

Let f : (X, T) → (Y, S) be a fuzzy pre-(γ, β)-continuous, fuzzy pre-(γ, β)-g continuous and fuzzy pre-(γ, β)-g closed mapping. Then the following statements hold:

(a) If f is injective and (Y, S) is a fuzzy pre-β-T½ space, then (X, T) is a fuzzy pre-γ-T½ space.

(b) If f is surjective and (X, T) is a fuzzy pre-γ-T½ space, then (Y, S) is a fuzzy pre-β-T½ space.

Proof:

(a) Let λ be a fuzzy pre-γ-g closed set in (X, T). Since f is fuzzy pre-(γ, β)-g closed, f (λ) is fuzzy pre-β-g closed. Since (Y, S) is a fuzzy pre-β-T½ space, f (λ) is fuzzy pre-β-closed. Since f is fuzzy pre-(γ, β)-continuous,

f^{-1} (f (λ)) is fuzzy pre-γ-closed. Hence (X, T) is a fuzzy pre-γ-T½ space.

(b) Let μ be a fuzzy pre-β-g closed set in (Y, S). Since f is fuzzy pre-(γ, β)-g continuous, $f^{-1}(\mu)$ is a fuzzy pre-γ-g closed set. Since (X, T) is a fuzzy pre-γ-$T_{1/2}$ space, $f^{-1}(\mu)$ is fuzzy pre-γ-closed. Therefore $\mu = f(f^{-1}(\mu))$ is a fuzzy pre-β-closed set. Hence (Y, S) is a fuzzy pre-β-$T_{1/2}$ space.

Proposition 4.7

Let f : (X, T) $\rightarrow$ (Y, S) be a fuzzy pre-(γ, β)-continuous injective mapping. If (Y, S) is a fuzzy pre-β-T_2 (resp. fuzzy pre-β-T_1) space then (X, T) is a fuzzy pre-γ-T_2 (resp. fuzzy pre-γ-T_1) space.

Proof:

Let (Y, S) be a fuzzy pre-β-T_2 space. Let λ_1, λ_2 be any two fuzzy sets in (X, T) such that $\lambda_1 \not\leq \lambda_2$. Then there exist fuzzy pre-β-open sets λ, μ in (Y, S) with $f(\lambda_1) \leq \lambda$ and $f(\lambda_2) \leq \mu$ such that $\lambda \not\leq \mu$. Then $\lambda \leq 1 - \mu$, which implies that $f^{-1}(\lambda) \not\leq f^{-1}(\mu)$. Now, $\lambda_1 \leq f^{-1}(\lambda)$ and $\lambda_2 \leq f^{-1}(\mu)$. Since f is fuzzy pre-($\gamma$, β)-continuous, $f^{-1}(\lambda)$ and $f^{-1}(\mu)$ are fuzzy pre-γ-open sets such that $f^{-1}(\lambda) \not\leq f^{-1}(\mu)$. Hence (X, T) is a fuzzy pre-γ-T_2 space. Similarly we prove the case of fuzzy pre-β-T_1 space.

5. FUZZY PRE-γ-NORMAL AND FUZZY PRE-γ-COMPACT SPACES.

Definition 5.1 A fuzzy topological space (X, T) is said to be fuzzy pre-γ-normal if for every fuzzy pre-γ-closed set λ and fuzzy pre-γ-open set μ in (X, T) such that $\lambda \leq \mu$, there exists a fuzzy set δ such that $\lambda \leq \gamma$-fp int(δ) $\leq \gamma$-fp cl(δ) $\leq \mu$.

Proposition 5.1 For any fuzzy topological space (X, T) the following statements are equivalent :

(a) (X, T) is fuzzy pre-γ-normal.

(b) For each fuzzy pre-γ-closed set λ and each fuzzy pre-γ-open set μ in (X, T) such that $\lambda \leq \mu$, there exists a fuzzy pre-γ-open set δ in (X, T) such that γ-fp cl (λ) $\leq \delta \leq \gamma$-fp cl (δ) $\leq \mu$.

(c) For each fuzzy pre-γ-g closed set λ and each fuzzy pre-γ-open set μ in (X, T) such that $\lambda \leq \mu$, there exists a fuzzy pre-γ-open set δ in (X, T) such that γ-fp cl (λ) $\leq \delta \leq \gamma$-fp cl (δ) $\leq \mu$.

Proof (a) $\Rightarrow$ (b) The Proof is trivial.

(b) ⇒ (c) Let λ be any fuzzy pre-γ-g closed set and μ be any fuzzy pre-γ-open set in (X, T) such that $\lambda \leq \mu$. Since λ is fuzzy pre-γ-g closed, γ-fp cl $(\lambda) \leq \mu$. Now, γ-fp cl (λ) is fuzzy pre-γ-closed and μ is fuzzy pre-γ-open in (X, T). By (b), there exits a fuzzy pre-γ-open set δ in (X, T) such that γ-fp cl $(\lambda) \leq \delta \leq \gamma$-fp cl $(\delta) \leq \mu$.

(c) ⇒ (a) The proof is trivial.

Proposition 5.2 Let (X, T) and (Y, S) be any two fuzzy topological spaces. If f : (X, T) → (Y, S) is a fuzzy pre-(γ, β)-homeomorphism and (Y, S) is fuzzy pre-β-normal, then (X, T) is fuzzy pre-γ-normal.

Proposition 5.3 Let f : (X, T) → (Y, S) be a fuzzy pre-(γ, β)- homeomorphism from a fuzzy pre-γ-normal space (X, T) onto a fuzzy topological space (Y, S). Then (Y, S) is fuzzy pre-β-normal.

Proof: Let λ be any fuzzy pre-β-closed set and μ be any fuzzy pre-β-open set in (Y, S) such that $\lambda \leq \mu$. Since f is fuzzy pre-(γ, β)-continuous, $f^{-1}(\lambda)$ is fuzzy pre-γ-closed and $f^{-1}(\mu)$ is fuzzy pre-γ-open in (X, T). Since (X, T) is fuzzy pre-γ-normal, there exists a fuzzy set δ in (X, T) such that

$$f^{-1}(\lambda) \leq \gamma\text{-fp int}(\delta) \leq \gamma\text{- fp cl}(\delta) \leq f^{-1}(\mu).$$

Now, $f(f^{-1}(\lambda)) = \lambda \leq f(\gamma\text{-fp int}(\delta)) \leq f(\gamma\text{-fp cl}(\delta)) \leq f(f^{-1}(\mu)) = \mu$.

That is, $\lambda \leq \beta$-fp int $(f(\delta)) \leq \beta$-fp cl $(f(\delta)) \leq \mu$. Therefore, (Y, S) is fuzzy pre-β-normal.

Definition 5.2 A collection $\{\lambda_i\}_{i \in J}$ of fuzzy pre-γ-open sets (resp. fuzzy pre-β-open sets) of fuzzy topological space (X, T) is called fuzzy pre-γ (resp. fuzzy pre-β)-covering of (X, T) if $1_X \leq \vee \lambda_i$.

A fuzzy topological space (X, T) is called fuzzy pre-γ (resp. fuzzy pre-β)-compact if every fuzzy pre-γ (resp. fuzzy pre-β)-cover of (X, T) has a finite subcover.

A collection $\{\lambda_i\}_{i \in J}$ of fuzzy pre-γ(resp. fuzzy pre-β)-open sets in (X, T) is called fuzzy pre-γ(resp. fuzzy pre-β)-cover of a fuzzy set μ in (X, T) if $\mu \leq \vee \lambda_i$.

Proposition 5.4 Let f : (X, T) → (Y, S) be an fuzzy pre-(γ, β) -continuous surjective function of a fuzzy pre-γ-compact space (X, T) onto a fuzzy topological space (Y, S). Then (Y, S) is fuzzy pre-β-compact.

Proposition 5.5 Let f : (X, T) → (Y, S) be a fuzzy pre-(γ, β)-open bijective function and (Y, S) be a fuzzy pre-β-compact space. Then (X, T) is fuzzy pre-γ-compact.

REFERENCES

1. Bin Shahna.A.S. On fuzzy strong semi continuity and fuzzy pre continuity, Fuzzy Sets and Systems., 44 (1991), 303-308.
2. Caldas. M., Jafari. S and Noiri. T. Notions via g-open sets, Kochi J.Math., 2 (2007), 43-50.
3. Chang.C.L. Fuzzy topological spaces, J.Math. Anal. Appl., 24 (1968), 182-190.
4. Sai Sundara Krishnan. G., Ganster. M. and Balachandran.K. Operation approaches on semi-open sets and applications, Kochi J.Math., 2 (2007), 21-33.
5. Ramadan.A.A., Abbas.S.E. and Abd el Latif.A.A. On fuzzy bitopological spaces in Sostak's sense, Commun. Korean. Math. Soc., 21 (2006), 865-877.
6. Singal.M.K. and Prakash. N. Fuzzy pre-open sets and Fuzzy pre-separation axioms, Fuzzy Sets and Systems, 44 (1991), 273-281.
7. Smets.P. The degree of belief in a fuzzy event, Inform. Sci., 25 (1981), 1-19.
8. Sugeno.M. An introductory survey of fuzzy control, Inform. Sci., 36 (1985), 59 - 83.
9. Uma.M.K., Roja.E. and Balasubramanian.G. On fuzzy pre-$T_{1/2}$ and fg pre-connected spaces, Mathematical Forum., Vol XV (2002-2003)
10. Uma.M.K., Roja.E. and Balasubramanian.G. On some stronger forms of fuzzy pre-continuous functions, Bull. Allah. Math. Soc., 17 (2002), 101-111
11. Zadeh.L.A.Fuzzy sets, Information and Control, 8 (1965), 338-353.