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Origin of Certain Generating Functions of The Charlier Polynomial $C_m(a;x)$ from The View Point of Lie–Algebra

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Abstract: In this paper generating functions for the Charlier polynomial $Cm\ (a;x)$ are obtained with the help of the representation of a Lie-group $G(0,1)$ [2]

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1. Introduction

Let, $G(0,1)$ be a complex 4-dimensional Lie-group. This abstract group $G(0,1)$ consists of abl 4 x 4 matrices of the form

$$(1.1) \qquad g = \begin{bmatrix} 1 & ce^{\tau} & d & \tau \\ 0 & e^{\tau} & b & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad d, b, c, \tau \in C$$

where the group operation is matrix multiplication. We can introduce co-ordinates for the elements g in $G(0,1)$ by setting $g \equiv (d, b, c, \tau)$. The co-ordinates are valid over the entire group. The usual topology of G induces a topology in $G(0,1)$ and is simply connected (Pontrij- agin, ch, 8). $L[G(0,1)]$ can be identified with the space of 4x4 matrices of the form:

$$\alpha = \begin{bmatrix} 0 & x_2 & x_4 & x_3 \\ 0 & x_3 & x_1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, x_1, x_2, x_3, x_4 \in C$$

with Lie product $[\alpha, \beta] = \alpha\beta - \beta d$; $\alpha, \beta \in L\,[G(0,1)]$. The matrices

$$g^{*} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, g^{-} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$g^3 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \varepsilon = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

form a basis of Lie-algebra $L[G(0,1)] = g\,(0,1)$ with the commutation relations.

$$[g^3, g^+] = g^+, [g^3, g^-] = -g^-, [g^+, g^-] = -\varepsilon, [\varepsilon, g^+] = [\varepsilon, g^-] = [\varepsilon, g^3] = 0$$

where 0 is the 4 x 4 zero matrix.

The mapping $\alpha \to \exp\alpha$ is an analytic diffeomorphism of a nbd. of $\theta \in L(G)$ on to a nbd. of e in G(here θ is the additive identity of $L[G(0,1)]$ and e is the identity element of $G(0,1)$. So, the mapping defines a local one to one coordinate transformation in C^4.

Here,

$$\exp\tau g^3 = \begin{bmatrix} 0 & 0 & 0 & \tau \\ 0 & e^\tau & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \exp b g^+ = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & b & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\exp C g^- = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \exp d\varepsilon = \begin{bmatrix} 1 & 0 & d & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$(\tau, b, c, d \in C)$$

Let, ρ be the representation of g(0,1) on the complex vector space V and let

let $\quad J^+ = \rho(g^+), J^- = \rho(g^+), E = \rho(\varepsilon), J^3 = \rho(g^3)$

These operator obey the commutation relations

$$[J^+, J^-] = -E, [J^3, J^+] = J^+, [J^3, J^-] = -J^-$$

$[J^+, E] = [J^-, E] = [J^3, E] = 0$ where $[A, B] = AB - BA$ for the linear operators A and B on V.

We define the Casimir operator $C_{0,1}$ on V by $C_{0,1} = J^+, J^- - EJ^3$. Let, ρ be the representation of g(0,1) satisfying the conditions : (i)ρ is irreducible (ii) each eigen value of J^3 has multiplicity equal to one. There is a countable basis for V consisting of eigen vectors of J^3. Such a representation ρ of g(0,1), for which $E \neq 0$, is isomorphic to the representation $R(w, m_0, \mu)$ defined for all $w, m_0, \mu \in C$ such that $\mu \neq 0$, $0 \leq \operatorname{Re} m_0 < 1$ and $w + m_0$ is not an integer. The spectrum of this representation is the set $S = \{m_0 + n\,;\, n \text{ is an integer}\}$ and the representation space V has a basis $\{f_m\}$, $m \in S$, so that

$$\text{(A)} \qquad \begin{cases} J^{+} f_m = \mu f_{m+1},\ J^{3} f_m = m f_m,\ J^{-} f_m = (m+w) f_{m-1} \\ E f_m = \mu f_m\ ,\ C_{0,1}\ f_m = \mu w f_m\ ; \end{cases}$$

where the differential operators J^{-}, J^{3} are given by

$$\text{(1.2)} \qquad \begin{aligned} J^{-} &= e^{-y} \left(x\frac{\partial}{\partial x} + \frac{\partial}{\partial y} \right) \\ J^{+} &= e^{y} \left(-\frac{\partial}{\partial x} + (1 - a/x) \right) \\ J^{3} &= \frac{\partial}{\partial y} \end{aligned}$$

such that

$$\text{(B)} \qquad \begin{cases} J^{-}\left[C_m(a\,;x)\, e^{my}\right] = m C_{m-1}\,(a\,;x)\, e^{(m-1)y} \\ J^{+}\left[C_m(a\,;x)\, e^{my}\right] = C_{m+1}\,(a\,;x)\, e^{(m+1)y} \\ J^{3}\left[C_m(a\,;x)\, e^{my}\right] = m C_m\,(a\,;x)\, e^{my} \end{cases}$$

From the relations (A) and (B), we have $\mu = 1$, $w = 0$. Thus the realization by differential operators yields a multiplier representation T of $G(0,1)$ whose Lie-algebra is $g(0,1)$.

2. Derivation of the generating functions:

It follows that the functions $f_m(x,y) = C_m(a;x)e^{my}$ form a basis for a realisation of the representation $R(0,\ m_{0}\ ,1)$ of $g(0,1)$. This representation of $g(0,1)$ by Lie-derivatives can be extended to a local multiplier representation of $G(0,1)$. If we denote '*cl*' the space of all entire analytic functions of x and y, the operators will uniquely define a multiplier representation T of $G(0,1)$ on '*cl*'. We compute the multiplier representation and obtain

$$\text{(2.1)} \qquad \left[T(\exp bJ^{+})\, f_m\right](x,t) = e^{bt}(1 - bt/x)^{a} f_m(x - bt, t) \quad \text{where } b = e^{y}$$

i.e. $t^{m} = e^{my}$

$$\text{(2.2)} \qquad \left[T(\exp cJ^{-})\, f_m\right](x,t) = f_m[x(t+c)/t\ ,\ t+c]$$

and

$$\text{(2.3)} \qquad \left[T(\exp \tau J^{3})\, f_m\right](x,t) = f_m(x, te^{\tau})$$

Also,

$$\text{(2.4)} \qquad \left[T(\exp dE)\, f_m\right](x,t) = e^{df_m}(x,t)$$

Now, we have

$$\text{(2.5)} \qquad \begin{aligned} &T\left[(\exp bJ^{+})(\exp bJ^{-})(\exp \tau J^{3})(\exp dE) f_m\right] \quad (x,\, t) \\ &= e^{d+bt}\,(1 - bt/x)^{a} f_m\left[(x - bt)(1 + c/t)\,,\ (t+c)e^{\tau}\right] \end{aligned}$$

where

(2.6) $$g = (\exp bJ^{+})(\exp cJ^{-})(\exp \tau J^{3})(\exp dE) \in G(0,1).$$

The matrix elements of this local representation with respect to the basic f_m are uniquely determined by $(R(0,m_o,1)$ and we obtain the relation.

(2.7) $$[T(g)f_{m_0+k}](x,t) = \sum_{\ell=-\infty}^{\infty} A_{\ell k}(g)\, f_{m_{0}+\ell}(x,t),\ k = 0, \pm 1; \pm 2, \ldots.$$

Thus we have

(2.8) $$e^{d+bt}(1-bt/x)^{a}(t+c)^{m_0+k}\, e^{(m_0+k)\tau}\, C_{m_0+k}[a;(x-bt)(1+c/t)].$$
$$= \sum_{\ell=-\infty}^{\infty} A_{\ell}, m-m_0 \quad (g) t^{m_0+\ell} \quad C_{m_0+\ell} \quad (a;x) \text{ where } k = m - m_0$$

The matrix elements $A_{\ell k}(g)$ are given by

(2.9) $$A_{\ell k}(g) = \frac{\exp[d+(m_o+k)\tau]\Gamma(m_o+k+1)\, c^{k-\ell}\, {}_1F_1}{(k-\ell)!\,\Gamma(m_o+\ell+1)}(-m_o-\ell; k-\ell+1; -bc)$$

for $k \geq \ell$.

and

(2.10) $$A_{\ell k}(g) = \frac{\exp[d+(m_o+k)\tau](b)^{\ell-k}\, {}_1F_1}{(\ell-k)!}(-m_o-k; \ell-k+1; -bc) \text{ for } \ell \geq k$$

Thus our result becomes

(2.11) $$e^{bt}(1-bt/x)^{a}(t+c/t)^{m}\, Cm[a;(x-bt)(1+c/t)]$$
$$= \sum_{n=0}^{\infty} \binom{m}{n} c^{m}\, {}_1F_1 \quad (-m_o+n; n+1; -bc) \quad C_{m-n}(a;x)\, t^{-n}$$

where $k-\ell = n$ with $|bt/x| < 1$, $|c/t| < 1$

This gives a new class of generating functions.

Also, we obtain

(2.12) $$e^{bt}(1-bt/x)^{a}(t+c/t)^{m}\, c_m[a;(x-bt)(1+c/t)].$$
$$= \sum_{n=0}^{\infty} \frac{b^n}{n!}\, {}_1F_1 \ (-m; n+1; -bc)\, c_{m+n}(a;x) \ t^{m} \quad \text{where } \ell - k = n$$

with $|b\,t/x| < 1$, $|c/t| < 1$

which gives another class of new generating functions.

3. Applications

(i) If we take $b=0$, $c=-1$ in (2.11) we have

$$(1-1/t)^{m}\, C_m\left[a; x\left(1-\frac{1}{t}\right)\right] = \sum_{n=0}^{\infty}(-1)^{n} \binom{m}{n} \quad C_{m-n} \ (a;x)\, t^{-n}$$

Replacing $1/t$ by t

$$+\frac{N^n N!}{R!}\sum_{n=rT+1}^{S+K}\frac{\rho^n}{(N+S-n)R^{n-(1-D)R-DT}\left(\frac{n!}{R!}\right)^D}\prod_{k=1}^{r-1}\left\{\frac{(R+k)^{DT}}{\left(1+\frac{\mu_k}{R_\mu}\right)^T}\right\}\frac{(R+r)^{(n-rT)D}}{\left(1+\frac{\mu_r}{R_\mu}\right)^{(n-rT)}}$$

The average number of failed units in the system is obtained by

$$(17)\quad E(N)=\sum_{n=0}^{S+K} n\,P_n$$

$$=\sum_{n=0}^{R}\frac{N^n\rho^n}{(n-1)}P_n+\frac{1}{R!}\sum_{n=R+1}^{S}\frac{N^n\rho^n n}{R^{(1-D)(n-R)}\left(\frac{n!}{R!}\right)}P_0+$$

$$+\frac{N^S N!}{R!}\sum_{n=S+1}^{T}\frac{n\rho^n}{(N+S-n)R^{(1-D)(n-R)}\left(\frac{n!}{R!}\right)^D}p_0$$

$$+\frac{N^S N!}{R!}\sum_{n=mT+1}^{(m+1)T}\frac{n\rho^n}{(N+S-n)R^{n-(1-D)R-DT}\left(\frac{n!}{R!}\right)^D}\prod_{k=1}^{m-1}\left\{\frac{(R+k)^{DT}}{\left(1+\frac{\mu_k}{R\mu}\right)}\right\}\frac{(R+m)^{(n-mT)D}}{\left(1+\frac{\mu_m}{R\mu}\right)^{(n-mT)}}P_0$$

$$+\frac{N^n N!}{R!}\sum_{n=mT+1}^{S+K}\frac{n\rho^n}{(N+S-n)!\,R^{n-(1-D)R-DT}\left(\frac{n!}{R!}\right)^D}\prod_{k=1}^{r-1}\left\{\frac{(R+k)^{DT}}{\left(1+\frac{\mu_r}{R\mu}\right)^T}\right\}\frac{(R+r)^{(n-rT)D}}{\left(1+\frac{\mu_r}{R\mu}\right)^{(n-rT)}}P_0$$

Case II: $S<R$

The failure rate and the repair rate for the model in this case are:

$$(18)\quad \lambda_n=\begin{cases} N\lambda & 0<n<S\\ (N+S-n)\lambda & S\le n<R\\ (N+S-n)\left(\frac{R}{n+1}\right)^b\lambda, & R\le n<T\\ (N+S-n)\left(\frac{R+m}{n+1}\right)^b\lambda, & mT\le n<(m+1)T,\ 1\le m<r\\ (N+S-n)\left(\frac{R+r}{n+1}\right)^b\lambda, & rT\le n<S+K \end{cases}$$

and

$$(19)\quad \mu_n=\begin{cases} n\mu, & 0<n\le R\\ \left(\frac{n}{R}\right)^a R\mu, & R<n\le T\\ \left(\frac{n}{R+m}\right)^b (R\mu+\mu_m) & mT<n\le(m+1)T,\ 1\le m<r\\ \left(\frac{n}{R+r}\right)^b (R\mu+\mu_r) & rT<n\le S+K \end{cases}$$

Solving equations (3)-(12), the probabilities for different states are given by:

$$(13)\quad P_n=\begin{cases} \dfrac{N^n\rho^n}{n!}P_0 & 0<n\le R\\[2ex] \dfrac{N^n\rho^n}{R!\,R^{(1-D)(n-R)}\left(\dfrac{n!}{R!}\right)}P_0, & R<n\le S\\[2ex] \dfrac{N^S N!\,\rho^n}{R!(N+S-n)!\,R^{(1-D)(n-R)}\left(\dfrac{n!}{R!}\right)}P_0, & S<n\le T\\[2ex] \dfrac{N^S N!\,\rho^n}{R!(N+S-n)!\,R^{n-(1-D)R-DT}\left(\dfrac{n!}{R!}\right)^D}\prod_{k=1}^{m-1}\left\{\dfrac{(R+k)^{DT}}{1+\left(1+\frac{\mu_k}{R_\mu}\right)^T}\right\}\dfrac{(R+k)^{(n-rT)D}}{\left(1+\frac{\mu_m}{R_\mu}\right)^{(n-mT)}}P_0, & mT<n\le(m+1)T,\quad 1\le m<r\\[2ex] \dfrac{N^S N!\,\rho^n}{R!(N+S-n)!\,R^{n-(1-D)R-DT}\left(\dfrac{n!}{R!}\right)^D}\prod_{k=1}^{r-1}\left\{\dfrac{(R+k)^{DT}}{1+\left(1+\frac{\mu_k}{R_\mu}\right)^T}\right\}\dfrac{(R+r)^{(n-rT)D}}{\left(1+\frac{\mu_k}{R_\mu}\right)^{(n-rT)}}P_0, & rT<n\le S+K \end{cases}$$

where

$$(14)\qquad \rho\frac{\lambda}{\mu}\ D=a+b\,.$$

In order to determine the value of P_0, we use the normalizing condition.

$$(15)\qquad \sum_{n=0}^{S+K}P_n=1$$

Then we obtain

$$(16)\quad P_0^{-1}=\sum_{n=0}^{R}\frac{N^n\rho^n}{n!}+\frac{1}{R!}\sum_{n=R+1}^{S}\frac{N^n\rho^n}{R^{(1-D)(n-R)}\left(\frac{n!}{R!}\right)}+$$

$$+\frac{N^nN!}{R!}\sum_{n=S+1}^{T}\frac{\rho^n}{(N+S-n)R^{(1-D)(n-R)}\left(\frac{n!}{R!}\right)^D}+$$

$$+\frac{N^nN!}{R!}\sum_{m=mT+1}^{(m+1)T}\frac{\rho^n}{(N+S-n)R^{n-(1-D)R-DT}\left(\frac{n!}{R!}\right)^D}\prod_{k=1}^{m-1}\left\{\frac{(R+k)^{DT}}{\left(1+\frac{\mu_k}{R_\mu}\right)}\right\}\frac{(R+r)^{(n-mT)D}}{\left(1+\frac{\mu_r}{R_\mu}\right)^{(n-mT)}}$$

Here a and b are the parameters that indicate the degree to which repair rate and failure rate respectively affected by state of system.

The Chapman-Kolmogorov steady-state equations developed by the model are given as:

(3) $-N\lambda P_0 + \mu P_1 = 0,$

(4) $-[N\lambda + n\mu]\, P_n + N\lambda\, P_{n-1} + (n+1)\,\mu\, P_{n+1} = 0, \qquad 1 \le n < R$

(5) $-\left[N\left(\frac{R}{R+1}\right)^b \lambda + R\mu \right] P_R + N\lambda\, P_{R-1} + \left(\frac{R+1}{R}\right) R\mu\, P_{R-1} = 0$

(6) $-\left[N\left(\frac{R}{n+1}\right)^b \lambda + \left(\frac{n}{R}\right)^a R\mu \right] P_n + N\left(\frac{R}{n}\right)^b \lambda\, P_{n-1} + \left(\frac{n+1}{R}\right)^a R\mu\, P_{n+1} = 0, \qquad R < n < S$

(7) $-\left[(N+S-n)\left(\frac{R}{n+1}\right)^b \lambda + \left(\frac{n}{R}\right)^a R\mu \right] P_n + (N+S-n+1)\left(\frac{R}{n}\right)^b \lambda\, P_{n-1}.$

$+\left(\frac{n+1}{R}\right)^a R\mu\, P_{n+1} = 0 \qquad S \le n < T$

(8) $-\left[(N+S-T)\left(\frac{R+1}{T+1}\right)^b \lambda + \left(\frac{T}{R}\right)^a R\mu \right] P_T + (N+S-T+1)\left(\frac{R}{T}\right)^b \lambda\, P_{T-1}$

$+\left(\frac{T+1}{R+1}\right)^a (R\mu + \mu_1) P_{T+1} = 0,$

(9) $-\left[(N+S-n)\left(\frac{R+m}{n+1}\right)^b \lambda + \left(\frac{n}{R+m}\right)^a (R\mu + \mu_m) \right] P_n +$

$+(N+S-n+1)\left(\frac{R+m}{n}\right)^b \lambda\, P_{n-1} + \left(\frac{n+1}{R+m}\right)^a (R\mu + \mu_m)] P_{n+1} = 0$

$mT < n < (m+1)T, \quad 1 \le m < r$

(10) $-\left[(N+S-n)\left(\frac{R+r}{n+1}\right)^b \lambda + \left(\frac{n}{R+r-1}\right)^a (R\mu + \mu_{r-1}) \right] P_n +$

$+(N+S-n+1)\left(\frac{R+r-1}{n}\right)^b \lambda\, P_{n-1} + \left(\frac{n+1}{R+r}\right)^a (R\mu + \mu_r) P_{n+1} = 0, \qquad n = rT$

(11) $-\left[(N+S-n)\left(\frac{R+r}{n+1}\right)^b \lambda + \left(\frac{n}{R+r}\right)^a (R\mu + \mu_r) \right] P_n +$

$+(N+S-n+1)\left(\frac{R+r}{n}\right)^b \lambda\, P_{n-1} + \left(\frac{n+1}{R+r}\right)^a (R\mu + \mu_r) P_{n+1} = 0$

$rT < n < S+K$

(12) $-\left(\frac{S+K}{R+r}\right)^b (R\mu + \mu_r)]\, P_{S-K} + (N-K+1)\left(\frac{R+r}{S+K}\right)^b \lambda\, P_{S+K-1} = 0$

production in the system. For model developing purpose, we have made in the following assumptions:

* The unit alternates in both states i.e. in operating state and repair (or failed) state.
* The standby spare units replace the failed operating units.
* The failure rate of the unit and repair rate of the permanent (additional) repairmen is λ and $\mu(\mu_i)$ $(i = 1,2, \ldots, r)$ respectively.
* The repair facility provides repair according to FCFS manner.
* The switch over times from failure to repair, from repair to standby and from standby to operating states are assumed to be negligible.
* When there are $n < T$ failed units, only R permanent repairmen are available to repair them.
* If there are $mT < n \leq (m+1)T$ failed units, there will be m special additional apart from R permanent repairmen in the system $(m = 1,2, \ldots, r-1)$.
* When $rT < n \leq S + K$ failed units, all the permanent and additional repairmen will be busy to provide service in the system.
* On completion of repair, the unit joins the set of standby units and treated as good as new ones.
* P_n represents the steady-state probability of n^{th} state $(n = 1,2, \ldots, S + K)$ while P_0 is the steady -state probability of empty.

3. Steady-state Equations and their Analysis

We consider two cases for analysis purpose, which are given as below

Case I: $R \leq S$. The birth death rates for the model are given by

$$(1) \qquad \mu_n = \begin{cases} N\lambda & 0 < n < R \\ N\left(\frac{R}{n+1}\right)^h \lambda, & R \leq n < S \\ (N+S-n)\left(\frac{R}{n+1}\right)^b \lambda, & S \leq n < T \\ (N+S-n)\left(\frac{R+m}{n+1}\right)^b \lambda, & mT \leq n < (m+1)T, 1 \leq m < r \\ (N+S-n)\left(\frac{R+r}{n+1}\right)^b \lambda, & rT \leq n < S+K \end{cases}$$

and

$$(2) \qquad \mu_n = \begin{cases} N\mu & 0 < n < R \\ \left(\frac{n}{R}\right)^a R\mu, & R < n < T \\ \left(\frac{n}{R+m}\right)^a (R\mu + \mu_m) & mT < n \leq (m+1)T, 1 \leq m < r \\ \left(\frac{n}{R+r}\right)^a (R\mu + \mu_r) & rT < n \leq S+K \end{cases}$$

machine repairmen problems were described by Either [2]. A cost function for the machine interference problem was developed by Moshe [17]. Wang and Hsu [19] carried out the cost analysis of machine repair problem with R non-reliable service stations. The M/G/I machine interference model with spares was studied by Gupta and Rao [5]. Jain [7] introduced diffusion approximation for (m.M) machine repair problem with spares and state dependent rates. M/M/R machine repair problem with spares and additional repairmen was also considered by Jain [8]. N-policy queueing system with finite source and warm spares was discussed by Gupta [3]. Jain and Dhyani [9] considered the transient analysis of M/M/C machine repair problem with spares. Jain considered the transient analysis of M/M/C machine repair problem with spares. Jain and Ghimire [10] investigated machine repair queueing system with non-reliable service stations and heterogeneous service discipline. Optimal repair/replacement policy for a general repair model was given by Jiang et al, [13]. They provided the repair cost-limit and the optimal average cost. Yakasai [20] developed a cost-off replacement policy for a component demanded by two parallel units by taking a minimum total cost function.

In the long queue of failed units, there may be some inconvenience due to which the failed units may be discouraged to join the queue. It means either they may balk or renege without being served. Some efforts are also made to analyze machine repair problem with balking and/or reneging, (m.M) machine repair problems with spares and reneging was investigated by Jain and Singh [12]. Ke and Wang [16] suggested cost analysis of the M/M/R machine repair problem with balking. reneging and server breakdowns. Recently Jain et al. [14] developed M/M/C/K/N machine repair problem with balking, reneging, spares and additional repairman. The two modes of failure machine repair problem with spares, reneging and additional repairman was also tackled by Jain et al. [15].

In the present paper we study a multi-component repairable system with spares and state-dependent rates by using birth-death techniques. The rest of the paper is organized in the following manner: The terminology of the model and notations used are given in section 2. In section 3, the balance equations in steady-state and their product obtain the optimal number of repairmen and spares, a heuristic approach is suggested in section 5. The conclusion and ideas for further development of the work are outlined in the last section 6.

2. The Model

We consider multi-component repairable system with N operating and S spare units. There is a provision of a repair facility consisting of R permanent and r additional repairmen. The system can accommodate only S+K units. The lifetime and repair time are state-dependent and are assumed to follow negative exponential distributions. There is provision of r special additional repairmen, which turn on one by one with the additional load of T failed units in order to maintain the regular