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A Cylindrically Symmetric Cosmological Model with Cosmic Cloud Strings in Bimetric Relativity

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Abstract: In this paper a cylindrically symmetric cosmological model is studied in the context of bimetric relativity taking the source as cosmic cloud strings and is found that the model does not exist in bimetric relativity

Key Words: Cylindrically symmetric cosmological model, Cosmic cloud strings, Bimetric relativity.

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1. Introduction:

Rosen[3] proposed a bimetric theory of relativity where there exist two metric tensors at each point of space-time–g_{ij} , which describes gravitation and background metric $-\gamma_{ij}$, which enters into the field equations directly with mater.

Accordingly, at each space-time point one has two line elements

$$ds^2 = g_{ij}\, dx^i\, dx^j$$

and

$$d\sigma^2 = \gamma_{ij}\, dx^i\, dx^j$$

This theory is based on a simple form of Lagrangian and has a simpler mathematical structure than that of the general relativity.

Deo[1] studied this model with the source perfect fluid distribution and found that the model does not exist in this theory.

Here a cylindrically symmetric non-static Einstein-Rosen cosmological model is studied with the source cosmic cloud strings and obtained the vacuum solutions.

2. Field Equations and Model

Field equations of bimetric relativity formulated by Rosen[3] are

(2.1) $$K_i^j = N_i^j - \tfrac{1}{2} N g_i^j = -8\pi k T_i^j$$

where

(2.2) $$N_i^j = \tfrac{1}{2}\gamma^{\alpha\beta}(g^{hj} g_{hi|\alpha})_{|\beta}$$

(2.3) $$N = N_\alpha^\alpha, \; k = (g/\gamma)^{\frac{1}{2}}$$

$$g = \det(g_{ij}) \text{ and } \gamma = \det(\gamma_{ij})$$

and a vertical bar (|) denotes the co'ariant differentiation with respect to γ_{ij}.

The energy-momentum tensor T_i^j for cosmic cloud strings is given by

$$T_i^j = T_{i\,\text{strings}}^j$$

(2.4) $$T_{i\,\text{strings}}^j = \rho v_i v^j - \lambda x_i x^j$$

Here ρ is the rest energy density for a cloud of strings with particle attached along the extension.

Thus $$\rho = \rho_p + \lambda,$$

where ρ_p is the particle energy density, λ is the tension density of the cloud strings, v^i is the four-vector representing the velocity of the cloud of particle and x^i is the four-vector representing the direction of anisotropy i.e. Z-axis.

So that $$v_4 v^4 = -1, x_3 x^3 = 1 \text{ and } v_i x^i = 0$$

Now consider the cylindrically symmetric non-static Einstein-Rosen cosmological model[2] given by

(2.5) $$ds^2 = e^{2(\alpha-\beta)}(dT^2 - (dR^3)) - R^2 e^{-2\beta} d\phi^2 - e^{2\beta} dZ^2$$

where α and β are functions of R and T and the convention is

$$x^1 = R, x^2 = \phi, x^3 = Z \text{ and } x^4 = T$$

The flat metric corresponding to (2.5) is

(2.6) $$d\sigma^2 = dT^2 - dR^2 - R^2 d\phi^2 - dZ^2$$

For this model non-vanishing Christoffel symbols are

(2.7) $$\Gamma_{12}^2 = \Gamma_{21}^2 = R^{-1}, \Gamma_{22}^1 = -R$$

Using equations (2.1) to (2.7) the field equations are

(2.8) $$R^{-2} \sin h(2\alpha) = 0$$

(2.9) $$\alpha'' + R^{-1}\alpha' - \ddot{\alpha} - R^{-2} \sin h(2\alpha) = 0$$

(2.10) $$\alpha'' + R^{-1}\alpha' - \alpha'' - 2\beta'' - 2R^{-1}\beta' + 2\ddot{\beta} = 8\pi k\lambda$$

(2.11) $$0 = 8\pi k\rho,$$

where $\alpha' = d\alpha / dR, \ \alpha'' = d^2\alpha / dR^2$ etc.

and $\alpha^{\cdot} = d\alpha / dT, \ \alpha^{\cdot\cdot} = d^2\alpha / dT^2$ etc.

By using equation (2.11)

$$\rho = 0$$

i.e. there is no contribution from energy density for a cloud of strings to the cylindrically symmetric non-static Einstein-Rosen cosmological model.

Thus the vacuum field equations are

(2.12) $$R^{-2} \sin h(2\alpha) = 0$$

(2.13) $$\alpha'' + R^{-1}\alpha' - \alpha^{\cdot\cdot} - R^{-2} \operatorname{Sin} h(2\alpha) = 0$$

(2.14) $$\alpha'' + R^{-1}\alpha' - \alpha^{\cdot\cdot} - 2\beta - 2R^{-1}\beta' + 2\beta'' = 0$$

From equation (2.12) we get

(2.15) $$\alpha = 0$$

Equation (2.14) and (2.15) gives

(2.16) $$\beta'' + R^{-1}\beta' - \beta^{\cdot\cdot} = 0$$

For solving equation (2.16)

Case I: Let us consider $\beta = H(r, t) + G(t)$

Then (2.16) gives the Bessel equation

(2.17) $$H'' + (1/R)\, H - H^{\cdot\cdot} = 0$$

representing cylindrical waves provided

(2.18) $$G^{\cdot\cdot} = 0$$

The Bessel equation (2.17) results

(2.19) $$H = a\, J_0(hR) \operatorname{Cos}(ht + c_1) + bY_0(hR) \operatorname{Sin}(ht + c_2)$$

where $J_0\,(hR)$ and $Y_0\,(hR)$ are Bessel functions of first and second kind of order zero respectively, h is the frequency and a, b, c_1, c_2 are arbitrary constants

On integration , (2.18) gives

(2.20) $$G = c_3 t + c_4$$

where c_3, c_4 are arbitrary constants of integration

Then

(2.21) $$\beta = a\, J_0(hR) \operatorname{Cos}(ht + c_1) + bY_0(hR) \operatorname{Sin}(ht + c_2) + c_3 t + c_4$$

Case II: Let us consider

$$\beta = H(r, t) + g(r)$$

In this case again we obtain equation (2.17) provided

(2.22) $$g'' + R^{-1} g' = 0$$

It gives us

(2.23) $$g = c_5 \log R$$

where c_5 is arbitrary constant of integration

Now the solution turns out to be

(2.24) $$\beta = a\, J_0(hR)\, \text{Cos}\,(ht + c_1) + bY_0(hR)\, \text{Sin}\,(ht + c_2) + c_5 \log R,$$

where constants could be found using initial as well as boundary conditions from the above equations.

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